

Change of Base Formula: $\log_b a = \frac{\log a}{\log b}$

6.6 Exponential and Logarithmic Equations

OBJ: To solve exponential and logarithmic equations

Intro: You invest \$1,000 in a mutual fund at an interest rate of 3%. How many years would it take for your money to reach \$2000?

$A(t) = A_0(1+r)^t$

$\frac{2000}{1000} = \frac{1000}{1000}(1+0.03)^t$

$\log_{1.03} 2 = \log_{1.03} (1.03)^t$

* Exponential Form \rightarrow Log

$\log_{1.03} 2 = t$

$\frac{\log 2}{\log 1.03} = t$

$23.4 \text{ yrs} \approx t$

$r = \frac{3}{100} = .03$ $t = ?$

Problem

Find the solution of $5^{2x} - 1 = 3$ by rewriting in logarithmic form.

Step 1: Add 1

$5^{2x} = 4$

Step 2: Exponential Form \rightarrow Log. Form

$\log_5 5^{2x} = \log_5 4$

Step 3: Change of Base Formula

$2x = \log_5 4$

Step 4: Divide 2 or Mult. $\frac{1}{2}$

$\frac{1}{2} \cdot 2x = \frac{1}{2} \log_5 4$

$x = \frac{\log_5 4}{2} \approx 0.43$

$x = ?$

Solve each exponential or logarithmic equation. Round the answer to the nearest hundredth.

1. $4^x - 8 = 3$

* Exp. Form \rightarrow Log. Form

$4^x = 8$

$\log_4 4^x = \log_4 8$

$x = \log_4 8$

$x = \frac{\log 8}{\log 4}$

$x \approx \frac{3}{2}$

2. $7 \log x = 21$

$\log x = 3$

$x = 10^3$

$x = 1,000$

* Log Form \rightarrow Exp

3. $5 + 2^{x+6} = 9$

$2^{x+6} = 4$

$\log_2 2^{x+6} = \log_2 4$

$x+6 = \log_2 4$

$x = \log_2 4 - 6$

$x = \frac{\log 4}{\log 2} - 6 = -4$

* Exp Form \rightarrow Log Form

4. $3^{2x-3} = 7$

* Exp. Form \rightarrow Log. Form

$\log_3 3^{2x-3} = \log_3 7$

$2x-3 = \log_3 7$

$2x-3 = \frac{\log 7}{\log 3}$

$2x = \frac{\log 7}{\log 3} + 3$

$2x = 4.771$

$x \approx 2.39$

5. $\log_5(4x-3) + 6 = 4$

$\log_5(4x-3) = -2$

$4x-3 = 5^{-2}$

$4x - \frac{3}{4} = \frac{1}{25}$

$\frac{1}{4} \cdot 4x = 3\frac{1}{25}$ or $\frac{76}{25} \cdot \frac{1}{4} = 4$

$x = \frac{19}{25}$

* Log Form \rightarrow Exp. Form

6. $2 \log 3 + \log 2x = 6$

$\log 3^2 + \log 2x = 6$

$\log 9 + \log 2x = 6$

$\log(9 \cdot 2x) = 6$

$\log(18x) = 6$

* Log Form \rightarrow Exp. Form

$18x = 10^6$

$\frac{18x}{18} = \frac{1,000,000}{18 \div 2}$

$x = \frac{500,000}{9}$

7. $\ln(6+x) - \ln(3-x) = 0$

$\ln\left(\frac{6+x}{3-x}\right) = 0$

$\frac{6+x}{3-x} = e^0$

$(3-x) \cdot \frac{6+x}{(3-x)} = 1 \cdot (3-x)$

$\frac{6+x}{6+2x} = 3$

* Write as single log.

* Log. Form \rightarrow Exp. Form

$\frac{2x}{2} = \frac{-3}{2}$

$x = \frac{-3}{2}$

Solve each equation. Round answers to the nearest hundredth when necessary.

1. $8^{2x} = 32$

2. $8 \log x = 16$

3. $4^{n-2} = 3$

4. $\log 3x = 2$

5. $\ln(x - 25) = 2$

6. $\log(x - 2) - \log(x + 3) = 1$

7. $4^n - 1 = 5$

8. $3 \log(1 - 2x) = 6$

9. $2 \ln x - \ln 3 = 1$

10. A parent increases a child's allowance by 10% each year. If the allowance is \$3 now, when will it reach \$15?

11. You deposit \$3000 into a bank account that pays 1.25% annual interest, compounded quarterly. How much interest does the account earn after 5 years?

12. An exotic bird population is decreasing at a rate of 0.02 per year. There are currently about 100,000 birds in the population. How many birds will there be in the population in 150 years?

Warm-Up

1. Write the logarithmic expression as a single logarithm: $\log a - \log ab$
2. Expand the logarithm: $\log 2x^3y$