6.2 The Natural Base e

OBJ: Define and use the natural base e, graph natural base functions

$y = ab^x$ Growth $b>1$; Decay $0<b<1$

Natural Number: $e \approx 2.718$ $\pi \approx 3.14$

Complete the table.

x	10	100	1000	10,000	100,000	1,000,000
$\left(1+\frac{1}{x}\right)^x$	$\left(1+\frac{1}{10}\right)^{10} \approx 2.59374$	2.7048	2.7169	2.7181	2.7183	2.7183

1. Rewrite each expression into simplest exponential form with positive exponents only.

a.) $e^{2} \cdot e^{9}$ — e^{11}

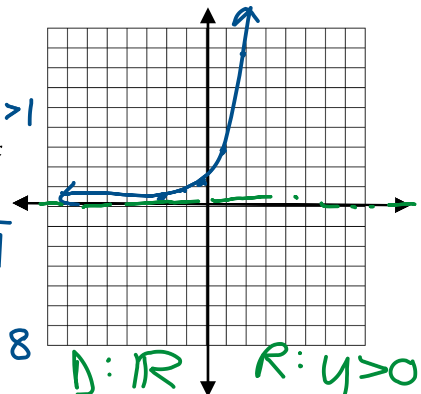
b.) $\frac{25e^{13}}{5e^{10}}$ — $= 5e^{3}$

c.) $(2e^{-3x})^5$ — $= 2^5 e^{-15x} = 32e^{-15x} = \frac{32}{e^{15x}}$

d.) $\sqrt{9e^{6x}}$ — $= 3e^{3x}$

e.) Graph $y = e^x$ Growth $b>1$

x	y
−2	0.14
−1	0.4
0	1
1	2.718
2	7.4



D: ℝ R: $y>0$

2. Tell whether each function represents exponential growth or exponential decay. What is the initial amount? What is the y-intercept? a.) $f(x) = 2.5e^x$ b.) $y = e^{-0.2x}$

a.) $b = 2.7$, $b>1$ Growth; $A_0 = 2.5$; y-int $= 2.5$

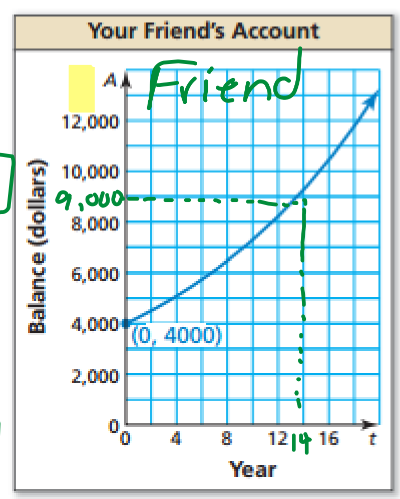
b.) $= \left(\frac{1}{e^{0.2}}\right)^x$; $0<b<1$ Decay; $A_0 = 1$; y-int $= 1$; DF $= \frac{1}{e^{0.2}}$

Exponential Function: $y = ab^x$	Exponential Growth/Decay: $A(t) = A_0(1 \pm r)^t$ (+ Increase, appreciate; − Decrease, depreciate)
Compound Interest: $A(t) = A_0\left(1+\frac{r}{n}\right)^{nt}$ — annually: $n=1$; semi-ann: $n=2$; quarterly: $n=4$; monthly: $n=12$; daily: $n=360$	Continuously Compounded Interest: $A(t) = Pe^{rt}$ ($P = A_0$ (Principal))

3. Suppose you invest \$7500 at an annual interest of 7% compounded continuously. How much will you have in the account in 10 years?

$r = \frac{7}{100} = .07$; $A(t) = Pe^{rt}$; $A(10) = 7500e^{.07(10)} \approx \$15,103.15$

4. You and your friend each have accounts that earn annual interest compounded continuously. Use the graph for the balance of your friend's account over time. The balance A (in dollars) of your account after t years can be modeled by $A = 3900e^{0.05t}$



a.) Which account has a greater principal? Me: \$3900; Friend: \$4000

b.) What is your interest rate? $r = 5\%$

c.) What is your balance after 10 years? $A(10) = 3900e^{0.05(10)} \approx \$6,430.01$

d.) Which account has a greater balance after 14 years?

Me: $A = 3900e^{0.05(14)} \approx \$7,853.64$

Friend: \$9,000

6.2 Natural Base of e HW Name______________________________

In Exercises 1–6, simplify the expression.

1. $e^2 \bullet e^5$ **2.** $e^{-3} \bullet e^8$ **3.** $\frac{12e^5}{36e^2}$ **4.** $\frac{15e^4}{3e^9}$ **5.** $\left(3e^{3x}\right)^2$ **6.** $\sqrt{16e^{10x}}$

In Exercises 7–9, tell whether the function represents *exponential growth* or *exponential decay*. What is the initial amount? What is the y-intercept?

7. $y = e^{4x}$ **8.** $y = e^{-x}$ **9.** $y = 4e^{-2x}$

10. You invest $4000 in an account to save for college.

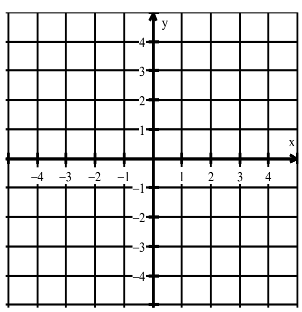
a. Option 1 pays 5% annual interest compounded semi-annually. What would be the balance in the account after 2 years?

b. Option 2 pays 4.5% annual interest compounded continuously. What would be the balance in the account after 2 years?

11. The price of a new home is $126,000. The value of the house appreciates 2% each year. How much will the home be worth in 10 years?

12. A car depreciates 10% each year. If you bought this car today for $5000, how much will it be worth in 7 years?

13. Graph $y=4^x$. Find domain and range.



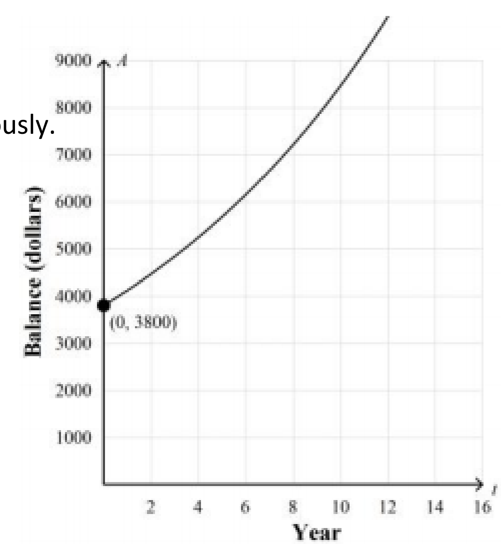
14. You and your friend have accounts that earn interest compounded continuously. The graph shows the account balance of your friend. Your balance is given by the equation $4100e^{.06t}$.

a.) Which account has a greater principal?

b.) What is your interest rate?

c.) What is your balance after 6 years?

d.) Which account has a greater balance after 6 years?



Warm-Up

What is the pattern in the table below? Write an equation to describe the data.

x	0	1	2	3
y	5	10	20	40