5.1 nth Roots and Radicals (Day 2)

OBJ: Solve for real solutions to find the nth roots

$\sqrt{9}$ vs.   x = ? Solve $x^2 = 9$

3

$x = \pm 3$

Check: $(3)^2 = 9$ ✓
$(-3)^2 = 9$ ✓

The nth Root	n is even	n is odd
$\sqrt[n]{a} = a^{\frac{1}{n} \leftarrow \text{index}}$	$a > 0$, Positive Root Ex. $\sqrt[4]{16} = 2$	$a > 0$, Positive Root Ex. $\sqrt[5]{32} = 2$
$\sqrt[n]{a^p} = a^{\frac{p}{n} \leftarrow \text{index}}$	$a < 0$, Imaginary Root Ex. $\sqrt[4]{-16} = 2i$	$a < 0$, Neg. Root Ex. $\sqrt[5]{-32} = -2$

Neg. exponent
↓ Take the reciprocal

1. Evaluate each expression without a calculator.

a.) $8^{4/3}$
$= (\sqrt[3]{8})^4$
$= (2)^4$
$= 16$

b.) $16^{-5/4} = \dfrac{1}{16^{5/4}}$
$= \dfrac{1}{(\sqrt[4]{16})^5}$
$= \dfrac{1}{(2)^5}$
$= \dfrac{1}{32}$

c.) $4^{5/2}$
$= (\sqrt{4})^5$
$= (2)^5$
$= 32$

d.) $9^{-1/2} = \dfrac{1}{9^{1/2}}$
$= \dfrac{1}{\sqrt{9}}$
$= \dfrac{1}{3}$

2. Simplify each radical expression.

a. $\sqrt{36x^4}$
$= 6^2 \sqrt{x \cdot x \cdot x \cdot x}$
$= 6x^2$

$\sqrt[2]{x^{4/2}} = x^2$

b. $\sqrt{162x^3}$
$2 \cdot 81$
$9 \cdot 9$
$3 \; 3 \; 3 \; 3$
$3x\sqrt{2} \cdot 3 \cdot 3 \cdot x \cdot x \cdot x$
$= 9x\sqrt{2x}$

c. $\sqrt[3]{-343x^9y^{12}}$
$7 \cdot 49$
$7 \cdot 7$
$= \sqrt[3]{-7 \cdot 7 \cdot 7 \cdot x^3 x^3 x^3 \cdot y^3 y^3 y^3 y^3}$
$= -7x^3y^4$

d. $\sqrt[4]{16x^{16}y^{20}}$
$\sqrt[4]{2 \cdot 8 \cdot 7 \cdot 2 \cdot x^4 x^4 x^4 x^4 \cdot y^4 y^4 y^4 y^4 y^4}$
$= 2x^4y^5$

3. Solve for the real solution(s). Approximate each solution to the nearest tenth when necessary.

x = ?
* even roots have 2 solutions

a.) $5x^3 = 320$
$\dfrac{5x^3}{5} = \dfrac{320}{5}$
$\sqrt[3]{x^3} = \sqrt[3]{64}$ odd
$x = \sqrt[3]{64}$
$x = 4$
1 Real Soltn.

b.) $\sqrt[4]{(x+3)^4} = \sqrt[4]{24}$ even
$x + 3 = \sqrt[4]{24}$
$x + 3 = 24^{1/4}$
$x + 3 = \pm 2.2$
$\;\;\;\; -3 \;\;\; -3$
$x = -3 \pm 2.2$
$x \approx -0.8, -5.2$

c.) $x^4 = \dfrac{\sqrt[4]{16}}{\sqrt[4]{625}}$ even
$x = \pm \dfrac{2}{5}$

d.) $\dfrac{1}{2}x^5 = 512 \cdot 2$
$\sqrt[5]{x^5} = \sqrt[5]{1,024}$ odd
$x = 4$

e.) $\sqrt[4]{(x+5)^4} = \sqrt[4]{16}$ even
$x + 5 = \sqrt[4]{16}$
$x \pm 5 = \pm 2$
$x = -5 \pm 2$
$x = -3, -7$

f.) $\sqrt[3]{(x-2)^3} = \sqrt[3]{-14}$ odd
$x - 2 = \sqrt[3]{-14}$
$x - 2 = -2.4$
$\;\; +2 \;\;\; +2$
$x = -0.4$

4.

One of the best used cars for teen insurance is a Mazda 3. The cost of a new Mazda 3 in 2015 was $16,945. The current value is $12,695. What is the rate of depreciation? If r represents the rate, t represents the number of years, C represents current value, and P represents original price, use the formula $C = P(1-r)^t$.

C
r = %
P

$12,695 = 16,945 \cdot (1-r)^4$
$\dfrac{12,695}{16,945} = \dfrac{16,945(1-r)^4}{16,945}$
$\sqrt[4]{0.75} = \sqrt[4]{(1-r)^4}$
$0.93 = 1 - r$
$\dfrac{-1 \quad\quad -1}{-.07 = -r}$
$r = .07$
$r = 7\%$

5.1 nth roots, Rational Exponents (Day 2)   Name_____

In Exercises 1–6, simplify each radical expression.

1. $\sqrt{81x^4}$

2. $\sqrt{121y^{10}}$

3. $\sqrt[3]{8g^6}$

4. $\sqrt[3]{125x^9}$

5. $\sqrt[5]{243x^5y^{15}}$

6. $\sqrt[3]{\dfrac{64x^9}{343}}$

In Exercises 7–12, evaluate the expression without using a calculator.

7. $36^{3/2}$

8. $16^{3/4}$

9. $(-32)^{2/5}$

10. $(-125)^{5/3}$

11. $256^{-5/4}$

12. $27^{-4/3}$

In Exercises 13–18, Solve for the real solution(s). Approximate to two decimal places when appropriate.

13. $49x^2 = 25$

14. $x^5 = -233$

15. $x^4 + 19 = 100$

16. $(x-6)^2 = 40$

17. $\frac{1}{5}x^4 = 125$

18. $\frac{1}{7}x^3 = -49$

19. The cost of an iPhone 7 in 2016 was $769. The current value is $549. What is the rate of depreciation?
If r represents the rate, t represents the number of years, C represents current value, and P represents original price,
use the formula $C = P(1 - r)^t$.

Warm-Up

Complete #1 in the notes.