5.1 nth Roots and Radicals

OBJ: To find the nth roots

a. $2^2 = 4$ $\sqrt{4} = 2$ $4^{1/2} = 2$

b. $2^3 = 8$ $\sqrt[3]{8} = 2$ $2 \cdot 2 \cdot 2$ $(8)^{\frac{1}{3}} = 2$

c. $2^4 = 16$ $\sqrt[4]{16} = 2$ $2 \cdot 2 \cdot 2 \cdot 2$ $16^{1/4} = 2$

d. $2^5 = 32$ $\sqrt[5]{32} = 2$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ $32^{1/5} = 2$

e. a^n $\sqrt[n]{a} = a^{1/n}$

The nth Root	n is even	n is odd
$\sqrt[n]{a} = a^{1/n}$	$a > 0$, Positive Root Ex. $\sqrt[4]{16} = 2$	$a > 0$, Positive Root Ex. $\sqrt[5]{32} = 2$
	$a < 0$, Imaginary Root Ex: $\sqrt[4]{-16} = 2i$	$a < 0$, Negative Root E $\sqrt[5]{-32} = -2$

index, radicand

1. Find the indicated real *n*th root(s) of *a*. $\sqrt[n]{a}$

a.) $n = 5, a = -243$ odd $\sqrt[5]{-243} = -3$ (a neg.)

Check: $-3 \cdot -3 \cdot -3 \cdot -3 \cdot -3 = -243$ ✓

b.) $n = 8, a = 256$ even $\sqrt[8]{256} = 2$ ($a > 0$)

Check: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 256$ ✓

c.) $n = 4, a = -16$ even $\sqrt[4]{-16} = 2i$ ($a < 0$)

Check: $2 \cdot 2 \cdot 2 \cdot 2 = 16$ ✓

d.) $n = 3, a = 125$ odd $\sqrt[3]{125} = 5$ ($a > 0$)

Check: $5 \cdot 5 \cdot 5 = 125$ ✓

e.) Explain why $\sqrt[4]{-81}$ has no real roots.

Since the index is even and radicand is positive there are no real roots.

2. Convert each radical expression to a number with a rational exponent. $\sqrt[n]{a}$ $a^{1/n}$

a. $\sqrt[3]{x} = x^{1/3}$ (index, 1/3 index) b. $\sqrt[3]{x^2} = x^{2/3}$ (2 exponent, 3 index)

$\sqrt[n]{x} = x^{1/n}$ $x^{\frac{1}{n}} = \sqrt[n]{x}$

$\sqrt[n]{x^p} = x^{p/n}$ (p exp., n index) $x^{\frac{p}{n}} = \sqrt[n]{x^p}$ or $(\sqrt[n]{x})^p$

3. Convert each number with a rational exponent to a radical expression. $a^{1/n}$ $\sqrt[n]{a}$

a. $8^{1/3} = \sqrt[3]{8}$ b. $8^{2/3} = \sqrt[3]{8^2}$ or $(\sqrt[3]{8})^2$

4. Evaluate each expression without a calculator.

a.) $64^{4/3} = (\sqrt[3]{64})^4 = (4)^4 = 256$

b.) $16^{-5/4}$ (Neg exp.) $= \frac{1}{16^{5/4}} = \frac{1}{(\sqrt[4]{16})^5} = \frac{1}{(2)^5} = \frac{1}{32}$

c.) $4^{5/2} = (\sqrt{4})^5 = (2)^5 = 32$

d.) $9^{-1/2}$ (Neg exp.) $= \frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

e.) $81^{3/4} = (\sqrt[4]{81})^3 = (3)^3 = 27$

f.) $1^{7/8} = (\sqrt[8]{1})^7 = (1)^7 = 1$

5. Approximate each expression using a calculator to two decimal places.

a.) $10^{2/5} \approx 2.51$

b.) $\sqrt[3]{11^2} = 11^{2/3} \approx 4.95$

6. The cost of a new Ford Focus in 2015 was $22,432 (P). The current value is $12,900 (C). What is the rate of depreciation (rate at which the car loses value)? If *r* represents the rate, *t* represents the number of years, *C* represents current value, and *P* represents original price, use the formula $r = 1 - \sqrt[t]{\frac{C}{P}}$. ($t = 4$ yrs.) (r = % loses value)

$r = 1 - \sqrt[4]{\frac{12,900}{22,432}}$

$r = 1 - (12,900/22,432)^{\wedge}(1/4)$

$r \approx 0.129$

$r \approx 12.9\%$

5.1 nth roots, Rational Exponents (Day 1) Name_____

In Exercises 1–3, find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = -125$ **2.** $n = 2, a = 49$ **3.** $n = 3, a = 27$ 4. $n = 5, a = 32$

In Exercises 5–10, evaluate the expression without using a calculator.

5. $27^{1/3}$ 6. $16^{1/4}$ 7. $4^{3/2}$

8. $625^{3/4}$ 9. $(-1000)^{2/3}$ 10. $32^{1/5}$

In Exercises 11–14, evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

11. $\sqrt[5]{16,807}$ 12. $\sqrt[6]{15,625}$ 13. $12^{-1/3}$ 14. $6561^{5/4}$

15. One of the best used cars for teen insurance is a Honda Civic. The cost of a new Honda Civic in 2014 was $19,000. The current value is $13,800. What is the rate of depreciation? If *r* represents the rate, *t* represents the number of years, *C* represents current value, and *P* represents original price, use the formula $r = 1 - \sqrt[t]{\frac{C}{P}}$.

16. The cost of an iPhone 8 in 2017 was $699. The current value is $499. What is the rate of depreciation?
If *r* represents the rate, *t* represents the number of years, *C* represents current value, and *P* represents original price, use the formula $r = 1 - \sqrt[t]{\frac{C}{P}}$.

Warm-Up

Simplify

1. $\sqrt{9}$

2. 3^2

3. $\left(3^2\right)^{\frac{1}{2}}$