4.5 Rational Root Theorem (Day 2)

OBJ: To solve equations using the Rational Root Theorem

Rational Root Theorem

Intro: Solve $3x^2 + 5x - 2 = 0$

$x = ? \quad y = 0$

$a: \pm 3x \cdot x \quad C: \pm 1 \cdot 2$

$(3x - 1)(x + 2) = 0$

$3x - 1 = 0 \qquad \boxed{x = -2}$
$ +1 \; +1$

$\dfrac{3x}{3} = \dfrac{1}{3}$

$\boxed{x = \dfrac{1}{3}}$

$ax - c = 0$
$ +c \; +c$

$\dfrac{ax}{a} = \dfrac{c}{a}$

$\boxed{x = \dfrac{c}{a}}$

Example: What are the rational roots of $2x^3 - 5x^2 - 2x + 5 = 0$

$a: \pm 1 \; \pm 2 \qquad C: \pm 1 \; \pm 5$

Step 1: The constant, c, is __5__. List all factors of c: $\pm 1, \pm 5$

Step 2: The leading coefficient, a, is __2__. List all factors of a: $\pm 1, \pm 2$

Step 3: List all possible rational roots $\pm \dfrac{\text{factors of } c}{\text{factors of } a}$: $x = \dfrac{\pm 1, \pm 5}{\pm 1, \pm 2}$

Step 4: Test all possible rational roots. $x = \pm 1, \pm \dfrac{1}{2}, \pm 5, \pm \dfrac{5}{2}$

$P(-1) = 2(-1)^3 - 5(-1)^2 - 2(-1) + 5 = 0 \checkmark$

$\boxed{x = -1}$

Step 5: Use synthetic division until the polynomial is a quadratic.

$\begin{array}{r|rrrr} -1 & 2 & -5 & -2 & 5 \\ & & -2 & 7 & -5 \\ \hline & 2 & -7 & 5 & \boxed{0} \end{array}$

$2x^2 - 7x + 5$

Step 6: Factor the quadratic and solve.

$(x + 1)(2x^2 - 7x + 5) = 0$

$(x + 1)(2x - 5)(x - 1) = 0$

$\boxed{x = -1} \quad \boxed{x = \dfrac{5}{2}} \quad \boxed{x = 1}$

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1. $15x^3 - 32x^2 + 3x + 2 = 0$

2. $2x^3 - 9x^2 + 3x + 4 = 0$

$a: \pm 2 \cdot \pm 1 \qquad C: \begin{array}{l} \pm 2 \cdot \pm 2 \\ \pm 1 \cdot \pm 4 \end{array}$

$x = \dfrac{c}{a} = \dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

$x = \pm 1, \pm \dfrac{1}{2}, \pm 2, \pm 4,$

$P\left(-\dfrac{1}{2}\right) = 0 \checkmark \quad \boxed{x = -\dfrac{1}{2}}$

$\begin{array}{r|rrrr} -\dfrac{1}{2} & 2 & -9 & 3 & 4 \\ & & -1 & 5 & -4 \\ \hline & 2 & -10 & 8 & \boxed{0} \end{array}$

$\left(x + \dfrac{1}{2}\right)(2x^2 - 10x + 8) = \boxed{0}$

$\left(x + \dfrac{1}{2}\right) \cdot 2(x^2 - 5x + 4) = 0$

$2\left(x + \dfrac{1}{2}\right)(x - 1)(x - 4) = 0$

$\boxed{x = -1/2} \quad \boxed{x = 1} \quad \boxed{x = 4}$

4.5 Rational Root Theorem (Day2) HW   Name_____

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

1. $x^3 - 2x^2 - 5x + 6 = 0$

2. $x^3 + x^2 - 17x + 15 = 0$

3. $x^3 - 5x^2 - 2x + 24 = 0$

4. $2x^3 + 5x^2 + 4x + 1 = 0$

5. $5x^3 - 11x^2 + 7x - 1 = 0$

6. $2x^3 - 3x^2 - 8x + 12 = 0$

Warm-Up

Complete the intro. Problem

On the notes.

Divide $2x^4 - 15x^2 - 10x \div (x - 3)$