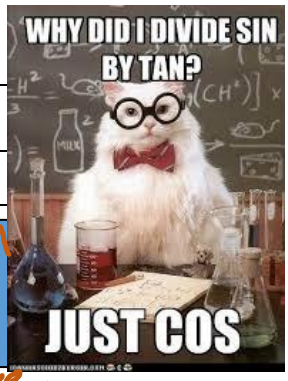
8.3 Modeling Periodic Behavior

HW p. 313 #1-15,19

cosine = sine starts = sine starts + $\frac{1}{4}$ · Pd.



General Sine and Cosine Equations

$y = A\sin B(x-C) + D$ ← stretch/shrink horizontally

$y = A\cos B(x-C) + D$ ← vertical shift / Axis of wave

$D = \frac{Max+Min}{2}$ ← amplitude

$A = \frac{Max-Min}{2}$ ← $B = \frac{2\pi}{Pd.}$ ← horiz shift

1. Write and equation for the graph.

a) $y = 2\sin 2(x-\frac{\pi}{6})+1$ or $y = 2\cos 2(x-\frac{5\pi}{12})+1$

$D = \frac{Max+Min}{2} = \frac{3+(-1)}{2}$

$D = 1$

$A = \frac{Max-Min}{2} = \frac{3-(-1)}{2}$

$A = 2$

$C = \frac{\pi}{6}$

$B = \frac{2\pi}{Pd.} = \frac{2\pi}{\pi} = 2$

scale: $\frac{\pi}{6}$ $Pd = \frac{7\pi}{6} - \frac{\pi}{6} = \pi$

cos starts $= \frac{\pi}{6} + \frac{1}{4}\cdot\pi = \frac{5\pi}{12}$

b)

$D = -1$

$A = 3$

$C = 1$

$B = \frac{2\pi}{Pd} = \frac{2\pi}{4} = \frac{\pi}{2}$

$y = 3\sin\frac{\pi}{2}(x-1)-1$

$y = 3\cos\frac{\pi}{2}(x-2)-1$

cos starts $= 1 + \frac{1}{4}\cdot 4 = 2$

$Pd = 5-1 = 4$

d)

$D = -1$

$A = 3$

$C = 1$

$B = \frac{2\pi}{Pd} = \frac{2\pi}{3}$

sin starts = cos starts $-\frac{1}{4}\cdot Pd$

$= 1 - \frac{1}{4}\cdot 3$

$= \frac{1}{4}$

$Pd = 4-1 = 3$

$y = 3\cos\frac{2\pi}{3}(x-1)-1$

$y = 3\sin\frac{2\pi}{3}(x-\frac{1}{4})-1$

$D = \frac{1}{2}$

$A = \frac{3}{2}$

$C = \frac{2\pi}{6} = \frac{\pi}{3}$

$B = \frac{2\pi}{Pd.} = \frac{2\pi}{\frac{2\pi}{3}} = 2\pi\cdot\frac{3}{2\pi} = 3$

cos starts $= \frac{\pi}{3} + \frac{1}{4}\cdot\frac{2\pi}{3} = \frac{\pi}{2}$

$Pd. = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$y = \frac{3}{2}\sin 3(x-\frac{\pi}{3})+\frac{1}{2}$

$y = \frac{3}{2}\cos 3(x-\frac{\pi}{2})+\frac{1}{2}$

2. Sketch the graph

a) $y = 2\cos\frac{1}{2}x - 2$

$D = -2$

$A = 2$

$C = 0$

$B = \frac{1}{2}$

$Pd. = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

b) $y = -4\sin\frac{1}{2}(x+\pi)$

$D = 0$

$A = |-4| = 4$

$C = -\pi$ Left

$B = \frac{1}{2}$

$Pd = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Max 4

Min −4

c) $y = 2\cos\frac{\pi}{2}(x-2)+1$

$D = 1$

$A = 2$

$C = 2$

$Pd = \frac{2\pi}{B} = 4$

Max 3

Min −1

d) $y = -3\sin\frac{1}{4}(x+\pi)-1$ reflect x-axis

$D = -1$

$A = |-3| = 3$

$C = -\pi$

$B = \frac{1}{4}$

$Pd. = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}} = 8\pi$

Marks = Pd ÷ 4 = 2π

Part 2

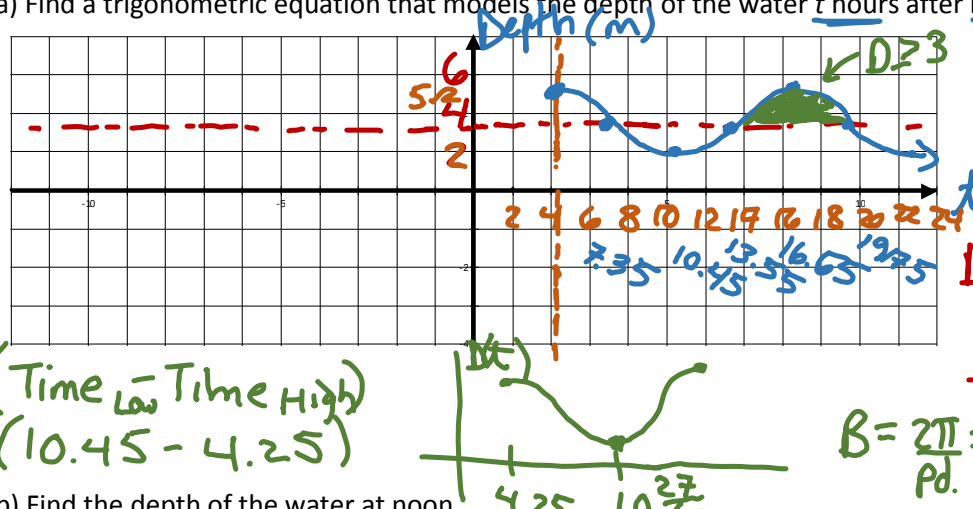
Marks = Pd ÷ 4 = 12.4 ÷ 4 = 3.1

she starts $= 4.25 - \frac{1}{4}\cdot(12.4) = 1.15$

1. The depth of water at the end of a pier varies with the tides throughout the day. Today the high tide occurs at 4:15 a.m. with a depth of 5.2 m. The low tide occurs at 10:27 a.m. with a depth of 2.0 m.

C: $t = 4\frac{15}{60} = 4.25$

a) Find a trigonometric equation that models the depth of the water t hours after midnight.



$A = \frac{Max - Min}{2} = \frac{5.2 - 2.0}{2}$

$A = 1.6$

$D = \frac{Max + Min}{2} = \frac{5.2 + 2.0}{2}$

$D = 3.6$

Pd = 2(Time Low − Time High)

Pd = 2(10.45 − 4.25)

Pd = 12.4

$B = \frac{2\pi}{Pd.} = \frac{2\pi}{12.4} = \frac{\pi}{6.2} = \frac{10\pi}{62} = \frac{5\pi}{31}$

b) Find the depth of the water at noon.

D(12) = ?

$D(t) = 1.6\cos\frac{5\pi}{31}(t - 4.25) + 3.6$

$= 1.6\sin\frac{5\pi}{31}(-1.15) + 3.6$

$D(12) = 1.6\cos\frac{5\pi}{31}(12 - 4.25) + 3.6 \approx$ 2.47 m

c) A large boat needs at least 3 m of water to moor at the end of the pier. During what time period after noon can it safely moor?

$D(t) \geq 3$

$t = ?$

$t > 12$

$D(t) = 1.6\cos\frac{5\pi}{31}(t - 4.25) + 3.6$

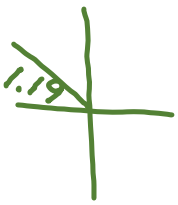
$3 = 1.6\cos\frac{5\pi}{31}(t - 4.25) + 3.6$

$-0.6 = 1.6\cos\frac{5\pi}{31}(t - 4.25)$

*Div. by 1.6

$\cos^{-1}(-0.375) = \cos^{-1}\cos\frac{5\pi}{31}(t - 4.25)$

ref ∠ ≈ 1.19



Q2: $\pi - 1.19 = 1.95$

Q3: $\pi + 1.19 = 4.33$

$1.95, 4.33, 2\pi + 1.95 = \frac{5\pi}{31}(t - 4.25)$

*3 separate eqtns. Mult by $\left(\frac{31}{5\pi}\right)$

$3.848 = t - 4.25$

$t = 8.1$ hrs

$\frac{1}{10}$ of 60 min

$t > 12$ → ~~$t = 8:06$ am~~

$8.5453 = t - 4.25$

$t = 12.8$ hrs

$\frac{8}{10}\cdot 60$

$t = 12:48$ pm

$16.2421 = t - 4.25$

$t = 20.5$ hrs.

$t = 8:30$ pm

Between 12:48 - 8:30 pm