7.4 Reference Angles

Day 1

$$x^2 + y^2 = 1$$

Sin 150°

$\left(\begin{array}{c} y \end{array}\right)$

30°   30°

30°

30°

Sin 30° $\left(\begin{array}{c} y \end{array}\right)$ → x

Sin 210°
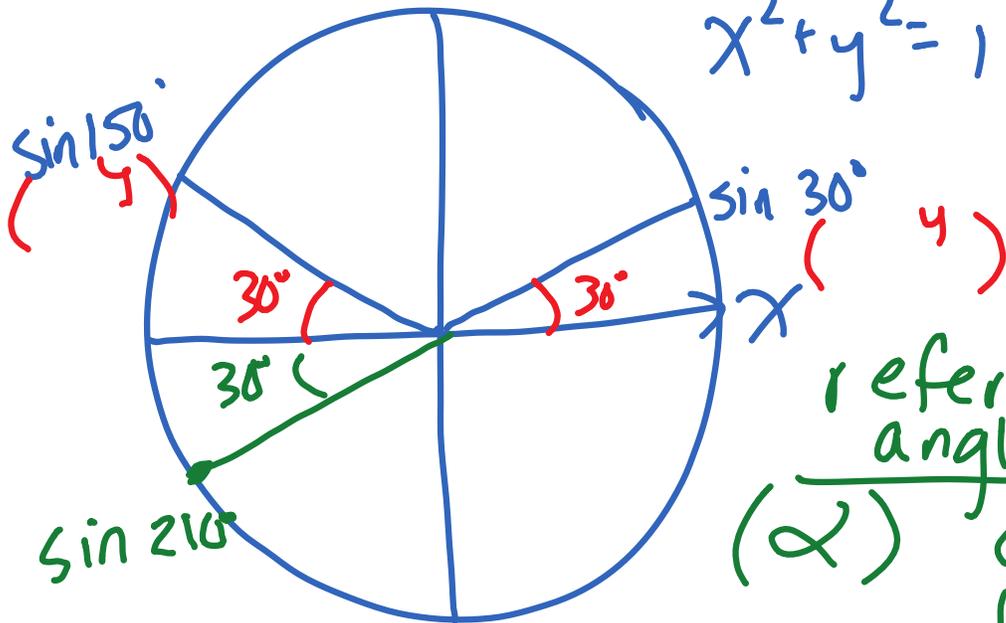
reference — a **positive**
angle

(α) **acute** angle
formed by the
terminal ray of θ
and the **x-axis**

① Express the following in
terms of a **reference** angle.
(acute, +, x-axis)

a.) $\sin 310° = \boxed{-\sin 50°}$

sin 50°

)50° 360°

C Sin 310°

b.) $\cos(-132°)$

$\boxed{= -\cos 48°}$

$\cot 132°$

140°   48°

$\begin{array}{r} 180° \\ -132 \\ \hline 48° \end{array}$
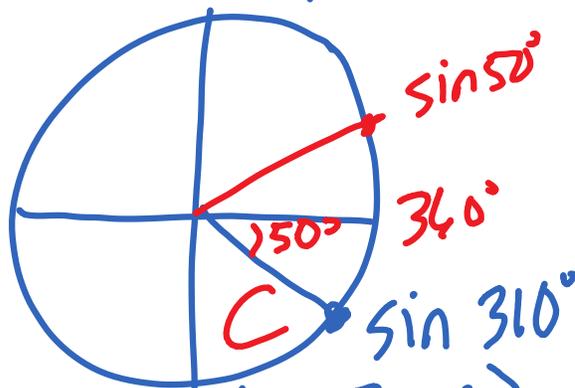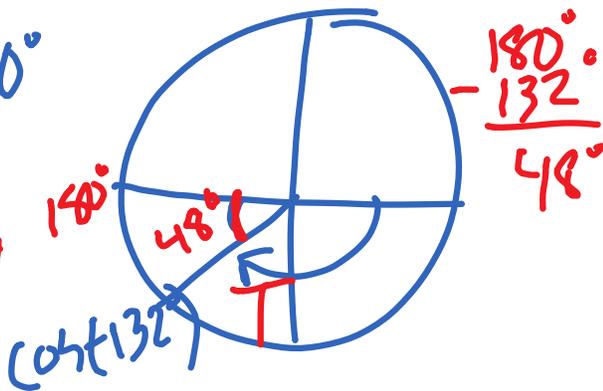
Day 2   SPECIAL ANGLES $90°, 45°, 60°, 30°$

$\dfrac{\pi}{2}, \dfrac{\pi}{4}, \dfrac{\pi}{3}, \dfrac{\pi}{6}$

30° - 60° - 90° △

45° - 45° - 90°
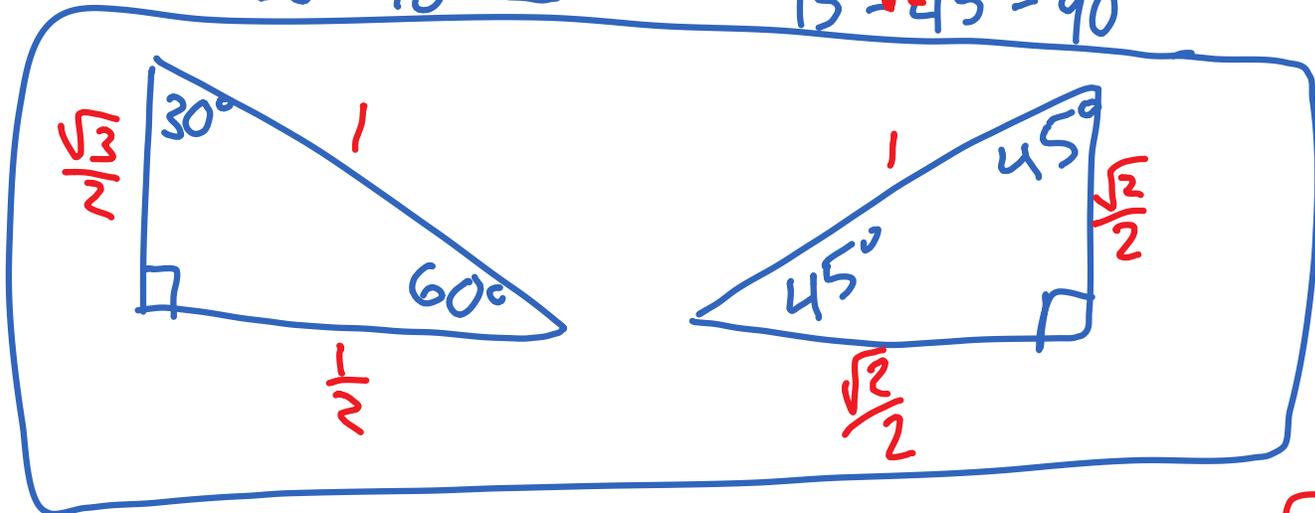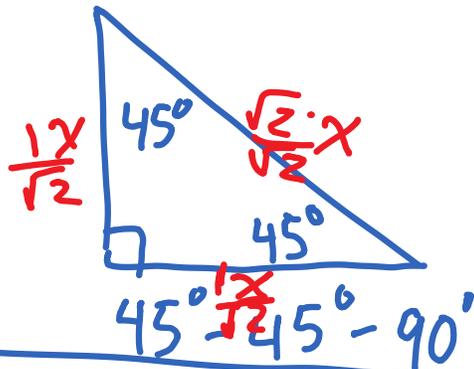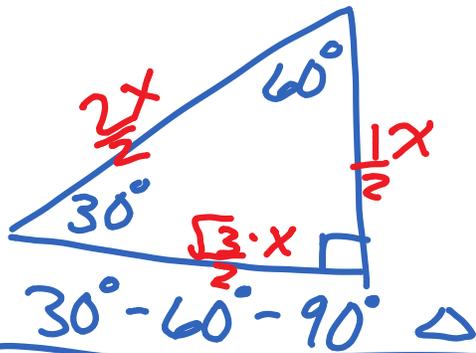
*Memorize

Ex. ① Find $\cos 30° = \dfrac{A}{H} = \dfrac{\frac{\sqrt{3}}{2}}{1} = \boxed{\dfrac{\sqrt{3}}{2}}$

② Find $\cos 120° = \dfrac{x}{r}$ or $\dfrac{A}{H} = \boxed{\dfrac{-1}{2}}$
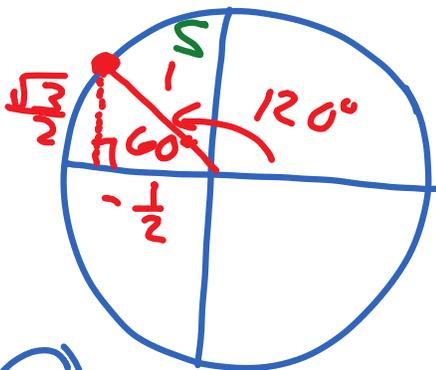
③ Find $\sin \dfrac{7\pi}{4} = \dfrac{y}{r}$ or $\dfrac{O}{H} = \dfrac{-\frac{\sqrt{2}}{2}}{1}$
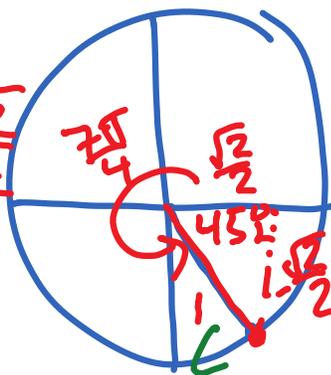
$= \boxed{\dfrac{-\sqrt{2}}{2}}$

SINE & COSINE GRAPHS

$y = \sin\theta$

Fundamental = 2π
Pd.

D: $\mathbb{R}$

R: $-1 \leq \sin\theta \leq 1$

Day 3

$y = \cos x$
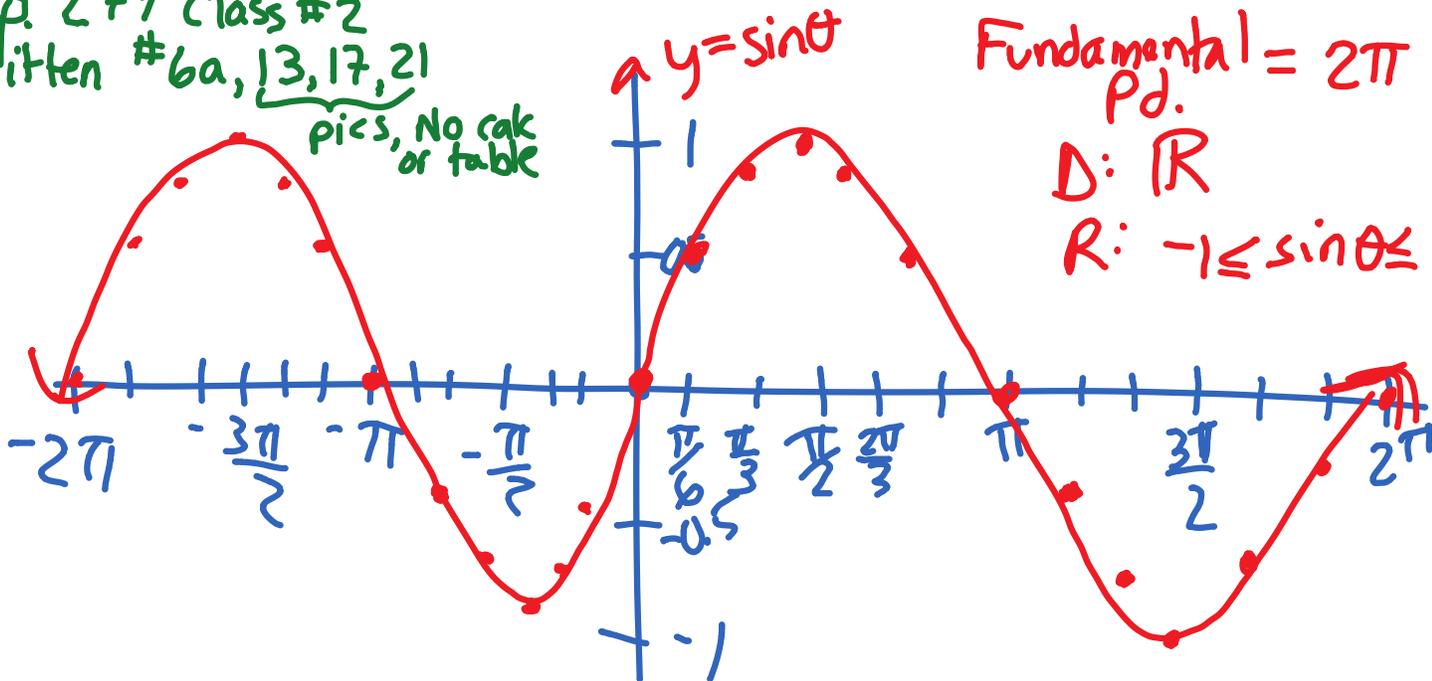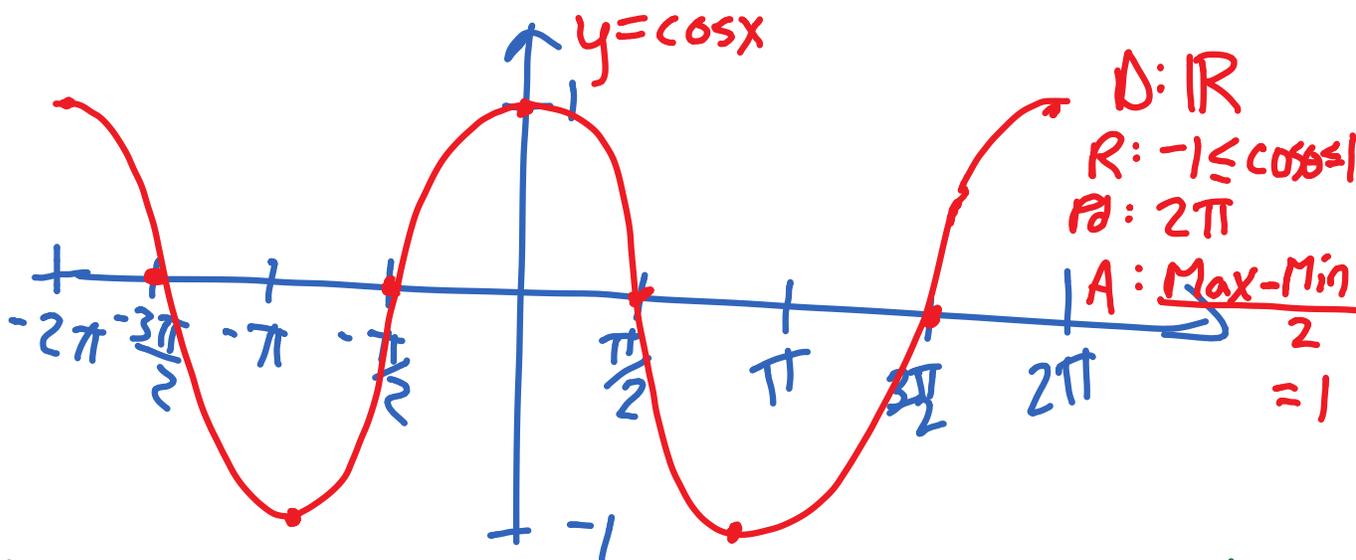
D: $\mathbb{R}$
R: $-1 \leq \cos\theta \leq 1$
Pd: 2π
A: $\dfrac{\text{Max}-\text{Min}}{2}$
$= 1$

* Sine is the cosine graph shifted left $\dfrac{\pi}{2}$

* $\sin\left(\theta + \dfrac{\pi}{2}\right) = \cos\theta$

Ex. ① Graph $y = 2\sin x$
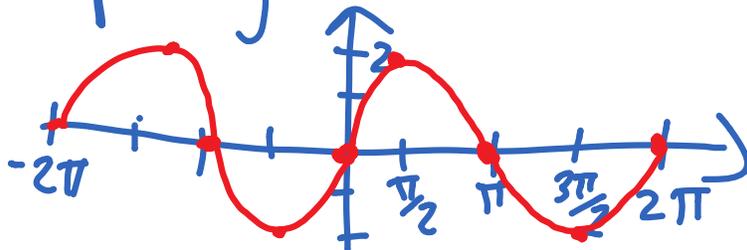
D: $\mathbb{R}$
R: $-2 \leq \sin\theta \leq 2$
Pd: 2π
A: $\dfrac{\text{Max}-\text{Min}}{2} = 2$

② Graph $y = 2\cos x$

LATITUDE

Indy 39°N

39°

r = 3,963 mi

0°
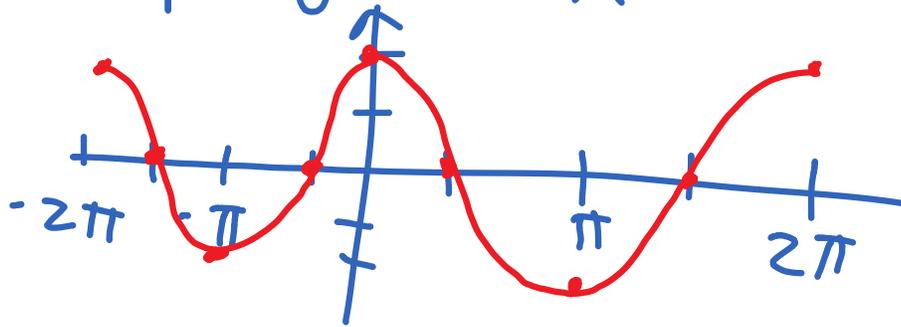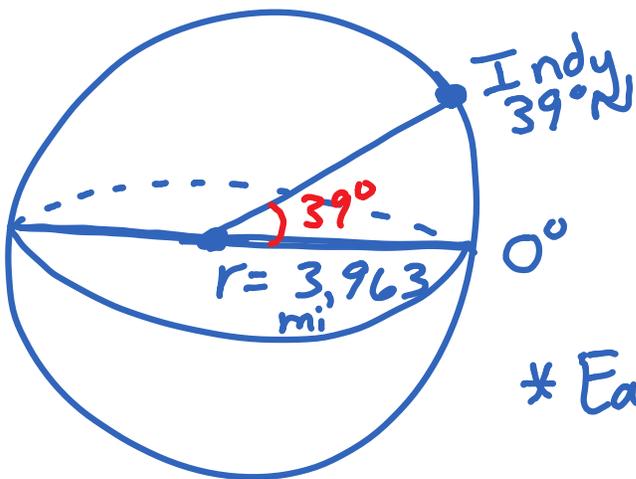
latitude - the shortest intercepted arc between a city the equator

* Earth's radius = 3,963 miles

Ex ① The latitude of Durham, NC is 36°N. How far from Durham is the North Pole?

N S=?

Durham

54°

36°
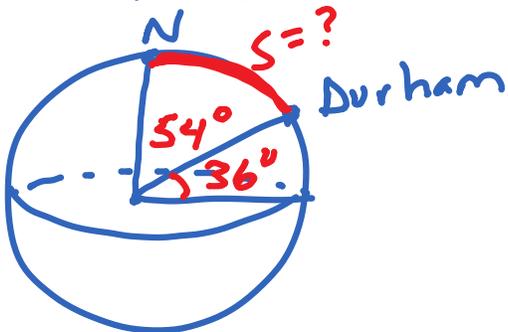
$$S = \frac{\theta}{360°} \cdot 2\pi r$$

$$S = \frac{54°}{360°} \cdot 2\pi (3963)$$

$$\boxed{S \approx 3,735 \text{ mi}}$$

② The latitude of Beijing is 39°55'N & the latitude of Perth, Australia is 31°58'S. How far apart are the 2 cities?

Beijing

$39°55'$

$31°58'$

S = ?

Perth
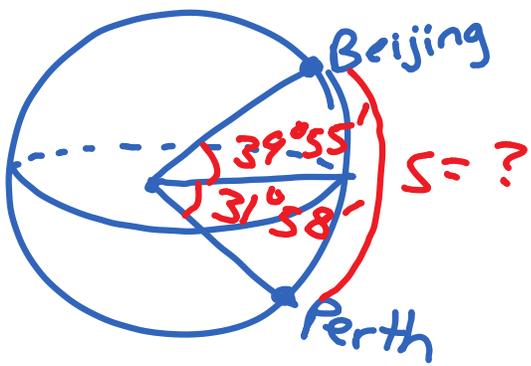
$39°53'$
$+ 31°60'$

$39°53'$
$+ 32°0'$
$71°53'$

or $71°\frac{53}{60}$
$= 71.88°$

$S = \dfrac{\theta}{360°} \cdot 2\pi r$

$S = \dfrac{71.88°}{360°} \cdot 2\pi (3963)$

$$S \approx 4{,}972 \text{ mi}$$

ROTATIONAL SPEED

r

$R = 3963$

$51°$ $R = 3963$

$39°$
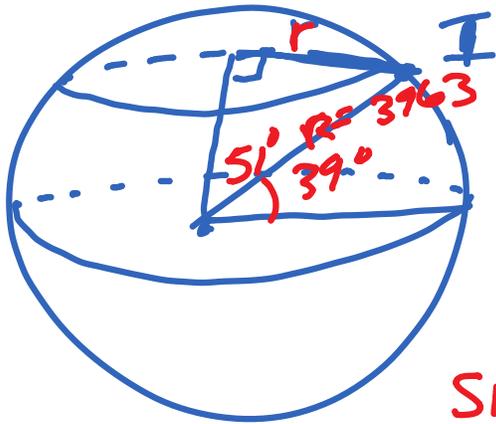
Earth's rotational Speed $= \dfrac{dist}{time} = \dfrac{2\pi r}{24 \text{ hrs}} = \dfrac{2\pi(3{,}963)}{24 \text{ hrs.}}$

$$\approx 1{,}038 \frac{mi}{hr}$$

$\sin 51° = \dfrac{r}{3963}$

$r = 3{,}079.83 \text{ miles}$

Indy's rotational Speed $\dfrac{2\pi r}{24 \text{ hrs}} = \dfrac{2\pi(3{,}079.83)\text{mi}}{24 \text{ hrs}} \approx \boxed{806.3 \frac{mi}{hr}}$