6.5 Parabolas

HW p.240 #2,~~3~~,6,7,11,15,18,26,~~27~~

Definition of a parabola:

Geometrically:

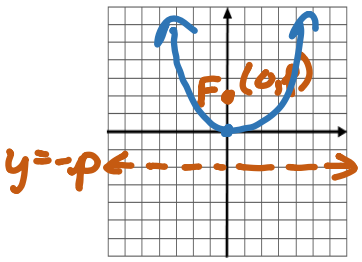
A parabola is the set of all points in the plane that are equidistant from a fixed point, called the focus, and a fixed line, called the directix.

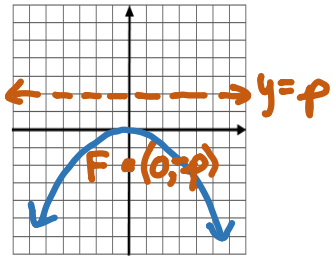
$\sqrt{(x-0)^2+(y-p)^2} = \sqrt{(x-x)^2+(y+p)^2}$

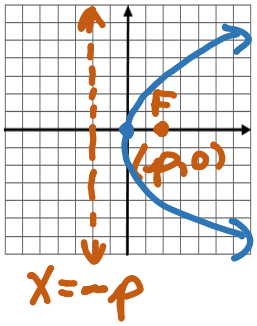
$y = \frac{1}{4p}x^2$

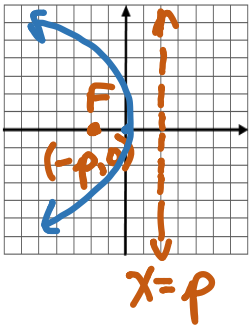
Algebraically:

a) Opens up: $y = \frac{1}{4p}x^2$ b) Opens down: $y = -\frac{1}{4p}x^2$ c) Opens right: $x = \frac{1}{4p}y^2$ d) Opens left: $x = -\frac{1}{4p}y^2$

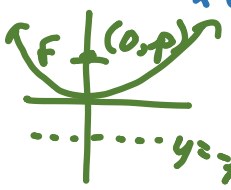








1. Find the focus and directix of the parabola for the equation $y = 2x^2$.



* opens up

$y = \frac{1}{4p}x^2$

$2 = \frac{1}{4p}$

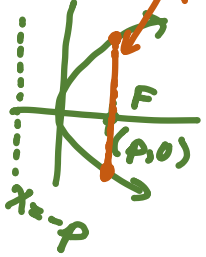
$8p = 1$

$p = \frac{1}{8}$

Focus: $(0, p)$ $(0, \frac{1}{8})$

Directrix: $y = -\frac{1}{8}$

2. Find the focus and directix of the parabola for the equation $x = \frac{1}{20}y^2$.Then sketch its graph.



* opens right

$x = \frac{1}{4p}y^2$

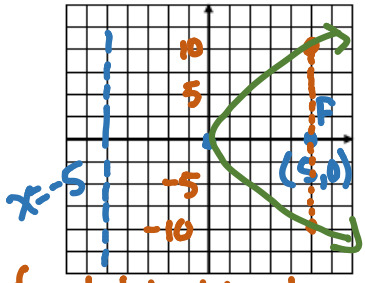
$\frac{1}{4p} = \frac{1}{20}$

$4p = 20$

$p = 5$

Focus: (5,0)

Directrix: x=-5



Length of Latus Rectum $= \left|\frac{1}{A}\right| = \left|\frac{1}{1/20}\right| = 20$ units

Latus Rectum: Line segment through the focus whose endpts fall on the parabola.

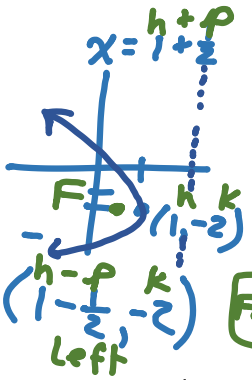
Formula: Length of Latus Rectum $= \left|\frac{1}{a}\right|$ * Makes graphing easier

3. Tell whether the parabola $x - 1 = -\frac{1}{2}(y+2)^2$ opens up, down. Right or left. Give the coordinates of the vertex and focus and the equation of the directix.

* opens left $\quad x = -\frac{1}{2}(y+2)^2 + 1 \rightarrow x = a(y-k)^2 + h$

$a = -\frac{1}{4p}$

* Opens up or down $y = a(x-h)^2 + k$



Vertex: (h, k)

$(1, -2)$

Focus: $-\frac{1}{4p} = -\frac{1}{2}$

$4p = 2$

$p = \frac{1}{2}$

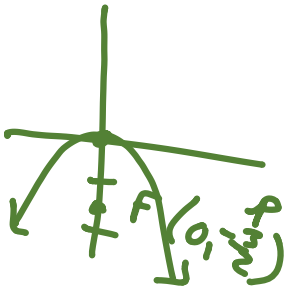
Focus: $(\frac{1}{2}, -2)$

Directrix: $x = h + p$

$x = 1 + \frac{1}{2}$

$x = 1\frac{1}{2}$

4. Find an equation of the parabola with vertex (0,0) and focus $\left(0, -\frac{3}{2}\right)$. Sketch the graph of the parabola.

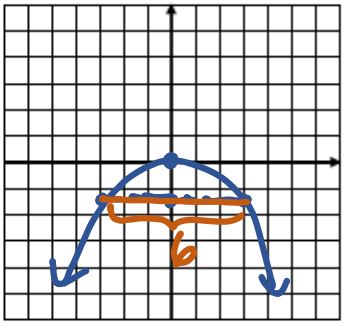


$y = -\frac{1}{4p}x^2$

$y = -\frac{1}{4(\frac{3}{2})}x^2$

$y = -\frac{1}{6}x^2$

Length of Latus Rectum $= \left|\frac{1}{-1/6}\right|$ 6 units



5. Find the vertex, focus, and directix of the parabola. Then sketch the graph of the parabola: $4y = x^2 - 8x + 12$.

$16 + 4y = x^2 - 8x + 16 + 12$

$16 + 4y = (x-4)^2 + 12$ * Sub 16

$4y = (x-4)^2 - 4$ * Div. 4

$y = \frac{1}{4}(x-4)^2 - 1$

Vertex: $(\frac{h}{4}, \frac{k}{-1})$

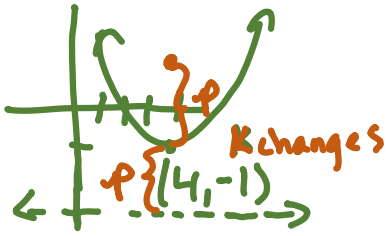
Focus: $(h, K+p)$

$\frac{1}{4p} = \frac{1}{4}$

$p = 1$

$(4, -1+1)$

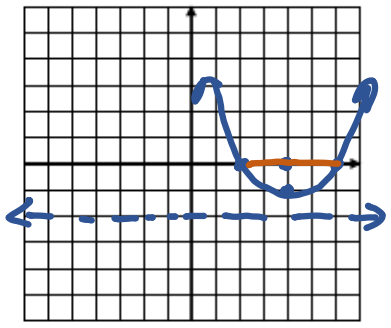
$(4, 0)$



Directrix: $y = k - p$

$y = -1 - 1$

$y = -2$



Latus Rectum: $\left|\frac{1}{a}\right| = \left|\frac{1}{1/4}\right|$

$= 4$ units

Warm-Up

Consider the equation: $y = 2x^2 + 8x - 3$.

1. The graph of the equation is called a _____.
2. Find the x- and y- intercepts of the graph of the equation.
3. Find the coordinates of the vertex of the graph of the equation.
4. Find the axis of symmetry of the equation. What is the domain and range of the equation?