5 – 7 Exponential Equations; Changing Bases Supplement

#1 A radioactive substance has a half-life of 1.4 years.

a) If there are 10 mg now, how much would you have in 7 years?

$A(t) = A_0(B)^{t/k}$

$A(7) = 10\left(\frac{1}{2}\right)^{7/1.4} \approx .3125 \text{ mg}$

b) How long does it take to decay to 4 mg?

$4 = 10\left(\frac{1}{2}\right)^{t/1.4}$

$.4 = \left(\frac{1}{2}\right)^{t/1.4}$

$\log_{\frac{1}{2}} .4 = \frac{t}{1.4}$

$1.4\left(\frac{\log .4}{\log \frac{1}{2}}\right) \approx t \approx 1.85 \text{ yrs}$

#2 Investment at 6% annual interest compounded daily – How long does it take to triple?

$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}$

$3 = \left(1 + \frac{.06}{360}\right)^{360t}$

$\log 3 = 360t \log\left(1 + \frac{.06}{360}\right)$

$t = \frac{\log 3}{360 \log\left(1 + \frac{.06}{360}\right)}$

#3 A $5,000 savings bond will double in 4 years.

a) Give a formula for A(t).

$A(t) = 5000(2)^{\frac{t}{4}}$ or

Rule of 72

$\frac{72}{R\%} = 4 \quad R = 18\%$

$A(t) = 5000(1 + .18)^t$

b) How long does it take to triple? 15,000

$3 = (2)^{t/4}$

$\log_2 3 = \frac{t}{4}$

$t = \left(\frac{\log 3}{\log 2}\right)4 \approx 6.339 \text{ yrs.}$

#4 Solve $\left(e^{x}\right)^{3} = 200$

$e^{3x} = 200$

$3x \ln e = \ln 200$

$x = \frac{\ln 200}{3}$

≈ 1.766

#5 Rule of 72: When will it double? Show that 69.3/r% is MORE accurate.

#6 $e^{2x} - e^{x} - 6 = 0$

Let $y = e^{x}$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3$ or $y = -2$

$3 = e^{x}$

$\ln 3 = x \ln e$

$x \approx 1.099$

$-2 = e^{x}$

$\ln -2 = x \ln e$