5.6 Laws of Logarithms

HW p.200 #'s 3 – 33 (x3), 37, 43, 45

Proofs: Let $a^b = x$ and $a^c = y$

* Rewrite in log. form

$\log_a x = b$ $\quad \log_a y = c$

$a^b \cdot a^c = xy$

$a^{b+c} = xy$

* Rewrite in log. form

$\log_a xy = b + c$

$\log_b a = \log_b a$

$(b^{\log_b a})^x = (a)^x$ * exp. form

$b^{x \log_b a} = a^x$

$\log_b a^x = x \log_b a$ * Raise to x power * Rewrite as log.

LAWS OF LOGARITHMS

Product and Sum (log prod = sum of 2 single logs)	1) $\log_a xy = \log_a x + \log_a y$
Quotient and Difference (log quot = diff. of 2 single logs)	2) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power (log(power) = exp. times log(base))	3) $\log_b a^x = x \log_b a$
Equality	4) $\log_b x = \log_b y$ iff $x = y$

1. Write each expression in terms of log M and logN.

a) $\log(MN)^2$ * Log of power

$= 2 \log(MN)$ * Log of prod.

$= 2[\log M + \log N]$

b) $\log \sqrt{\frac{M}{N}}$ * Log of power

$= \log \left(\frac{M}{N}\right)^{1/2}$

$= \frac{1}{2} \log\left(\frac{M}{N}\right)$ * log of quotient

$= \frac{1}{2}[\log M - \log N]$

2. Write as a rational # or as a single logarithm

a) $2 \ln 6 - \ln 3$

$= \ln 6^2 - \ln 3$

$= \ln 36 - \ln 3$

$= \ln\left(\frac{36}{3}\right)$

$= \ln 12$

b) $\frac{1}{3} \log 64 + 2 \log 5$

$= \log 64^{1/3} + \log 5^2$

$= \log 4 + \log 25$

$= \log(4 \cdot 25)$

$= \log 100 \longrightarrow = \log_{10} 10^2$

$= 2$

c) $\log 3 - \log 6 - \log 5$

$= \log\left(\frac{3}{6}\right) - \log 5$

$= \log\left(\frac{1/2}{5}\right)$

$= \log\left(\frac{1}{10}\right)$

$= \log_{10} 10^{-1}$

$= -1$

d) $\frac{1}{3}(2 \log_b M - \log_b N - \log_b P)$

$= \frac{1}{3}(\log_b M^2 - \log_b N - \log_b P)$ * $\frac{M^2}{N} \cdot \frac{1}{P}$

$= \frac{1}{3}\left[\log_b\left(\frac{M^2}{N}\right) - \log_b P\right]$

$= \frac{1}{3}\left[\log_b\left(\frac{M^2/N}{P/1}\right)\right] = \frac{1}{3}\log_b\left(\frac{M^2}{NP}\right)$

$= \log\left(\frac{M^2}{NP}\right)^{1/3}$

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e) $\log c + \log a + \log b + \log i + \log n$

log cabin

3. Review

a) Find $log_2 32$

$= \log_2 2^5$

$= 5$

b) $\ln e^4$

$= \log_e e^4$

$= 4$

4. Simplify

a) $\ln \frac{1}{e^3}$

$= \log_e \frac{1}{e^3}$

$= \log_e e^{-3}$

$= -3$

b) $e^{3\ln x}$

$= e^{\ln x^3}$

$= e^{\log_e x^3}$

$= x^3$

c) $10^{3+\log 3}$

$= 10^3 \cdot 10^{\log 3}$

$= 1000 \cdot 3$

$= 3000$

Express y in terms of x.

5. a) $\log y = 2\log x + \log 8$

$\log y = \log x^2 + \log 8$

$\log y = \log 8x^2$

$y = 8x^2$

b) $\log y = 3x - 1$

$10^{3x-1} = y$

$10^{3x} \cdot 10^{-1} = y$

$(10^3)^x \frac{1}{10} = y$ → $y = \frac{1}{10}(1000)^x$

6. **Solve for x.**

a) $log_6(x+1) + log_6 x = 1$

$\log_6 x(x+1) = 1$

$6^1 = x(x+1)$

$6 = x^2 + x$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3, 2$ (can't take $\log_6(-3+1)$)

$x = 2$

b) $log_2(x^2 + 8) = log_2 x + log_2 6$

$\log_2(x^2+8) = \log_2 6x$

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4, 2$