5.3 Exponential Functions

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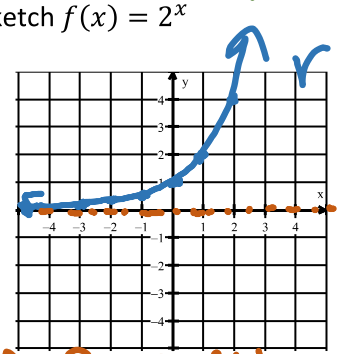
x	y
-3	$\frac{1}{2^3}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	2^0
1	2
2	4

Growth

Sketch $f(x) = 2^x$



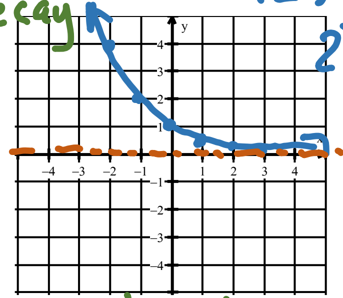
D: $\mathbb{R}$ R: $f(x)>0$

Z: None $(0,\infty)$

*Reflect thru y-axis

Sketch $f(x) = \left(\frac{1}{2}\right)^x$ or $(2^{-1})^x = 2^{-x}$

Decay



$0<b<1$

In General, for an **Exponential Function:** $f(x) = ab^x$

* Decay $0<b<1$
* Growth $b>1$

Linear Funct. $b=1$

$b<0$ Not an Exp Funct.

1. a) Given $f(0) = 3$ and $f(2) = 12$, determine the equation of the exponential function $f(x) = ab^x$.
   What two things to you need? __a__ and __b__.

$f(x) = ab^x \rightarrow$

$f(0) = ab^0$

$3 = a(1)$

$3 = a$

$f(x) = 3b^x$

$f(2) = 3b^2$

$12 = 3b^2$

$4 = b^2$

$\cancel{-2},\ 2 = b$

$\rightarrow \boxed{f(x) = 3(2)^x}$

b) Find f(-2).

$f(-2) = 3(2)^{-2}$

$\boxed{= 3/4}$

Exponential Growth & Decay	
Growth:	$A(t) = A_0(1+r)^t$ * increase, rate, growth
Decay:	$A(t) = A_0(1-r)^t$ * decrease, rate, depreciate
Growth OR Decay:	* $f(0)=5$, $f(2)=3$ $f(x) = ab^x$ (Decay: $0<b<1$) $A(t) = A_0\, b^{t/k}$ # yrs → t/k ← amount of time to double, triple, etc. b ← doubling, tripling, half-life

2. Describe the situation in a sentence or two. $A(t) = 100(3)^{\frac{t}{4}}$

School lunch costs \$100. The cost triples every 4 years.

3. Given the data table, form an exponential function for the data. Then determine P(t) for t = 24 hrs.

2^x

t hrs	P(t)
3	260
6	450
9	780

P(t) triples every 6 hrs.

$A(t) = A_0(3)^{t/6}$

$260 = A_0(3)^{3/6}$

$260 = A_0\sqrt{3}$

$\frac{260}{\sqrt{3}} = A_0$

$150 \approx A_0$

$\boxed{P(t) = 150(3)^{t/6}}$

$P(24) = 150(3)^{24/6} = \boxed{12,149}$

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Rule of 72

If a quantity is growing at <u>r</u> % per unit of time (year, day, month, etc…), then the <u>time to be doubled</u> is approximately <u>$72 \div r\%$</u>. ← not decimal form

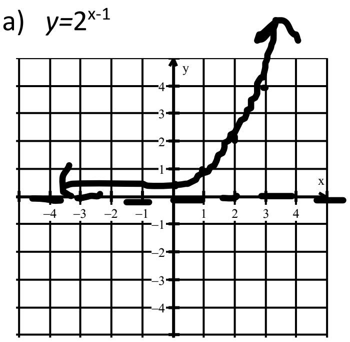
4. A bacteria colony increases <u>8%</u> (r) per day. Approximately how long does it take the colony to <u>double</u> in size?

$$t \approx 72 \div 8\% = \boxed{9 \text{ days}}$$

In summary for the formulas so far!	
Growth: Decay:	$A(t) = A_0(1+r)^t$ * rate increase, growth $A(t) = A_0(1-r)^t$ * rate depreciate, decay
Growth or Decay:	$f(x) = ab^x$ * $f(0)=1$ $f(2)=3$ $A(t) = A_0 b^{t/k}$ * half-life, double, triple
Rule of 72:	$72 \div r\%$ * rate and double
Compound Interest: Continuously Compounded Interest:	$A(t) = A_0\left(1+\frac{r}{n}\right)^{nt}$ * compound annually, quarterly, semi-ann., daily $A(t) = A_0 e^{rt}$ * compound continuously

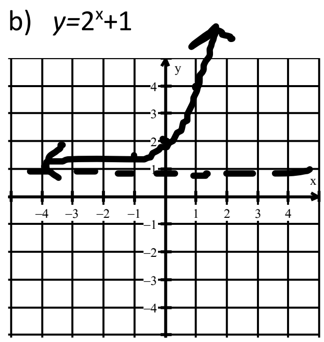
5. Sketch each graph. Find the domain, range, and zeros.

a) $y=2^{x-1}$



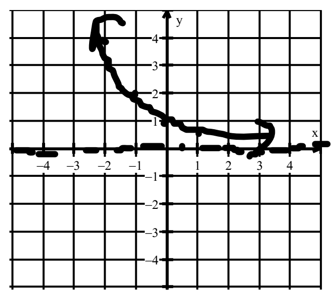
D: $\mathbb{R}$ R: $y>0$
Z: None

b) $y=2^x+1$



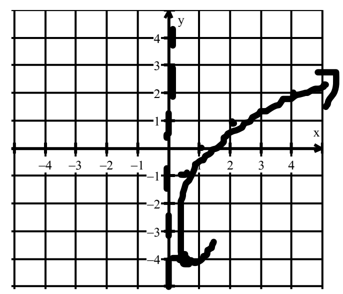
D: $\mathbb{R}$ R: $y>1$
Z: None

c) $y=2^{-x}$



D: $\mathbb{R}$ R: $y>0$
Z: None

d) $x=2^y$



D: $x>0$ R: $\mathbb{R}$
Z: 1