4.5 Inverse Functions

HW p.149 #1-23odd

30 → Mult 2 → Add 1 → 61

Go Backwards: 30 ← Div. 2 ← Sub. 1 ← 61 *Inverses undo one another

Examples: a) $F = \frac{9}{5}c + 32$ ← Inverses

$F - 32 = \frac{9}{5}C$

$\boxed{\frac{5}{9}(F - 32) = C}$

b) $V = x^3$ ← Inverses

$\sqrt[3]{V} = x$

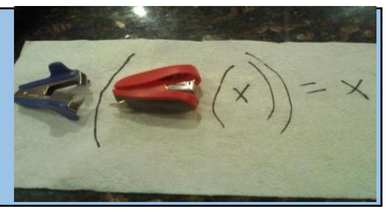
$f(x) = x^3$

$g(x) = \sqrt[3]{x}$

Definition: Inverse Function

Two functions $f(x)$ and $g(x)$ are inverse functions IFF: ⟺ (if and only if) 1) $f(g(x)) = x$

2) $g(f(x)) = x$



1. Determine if two functions are inverses:

a) $f(x) = 2x + 1, g(x) = \frac{x-1}{2}$

1. $f(g(x)) = 2\left(\frac{x-1}{2}\right) + 1$

$= x - 1 + 1$

$= x$ ✓

2. $g(f(x)) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$ ✓

∴ f & g are inverses

b) $f(x) = x^3, g(x) = \sqrt[3]{x}$

1. $f(g(x)) = (\sqrt[3]{x})^3 = x$ ✓

2. $g(f(x)) = \sqrt[3]{x^3} = x$ ✓

∴ f and g are inverses

c) If f(1)=4 and f(2)=5. Find:

$f^{-1}(4) = \boxed{1}$

$f^{-1}(f(2)) = f^{-1}(5) = \boxed{2}$

NOTE: Inverses reflect each other over the line $y = x$.

-4, 4 → $(\;\;)^2$ → 16   Not one - to - one function

Definition: One to One Function

A function that has an inverse

Def. function 1) Every x has EXACTLY 1 y.

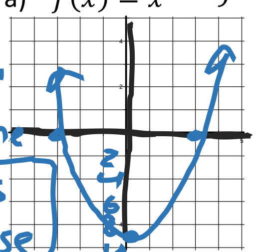
2) Every y has EXACTLY 1 x.   Must Pass vertical + horiz. Line Tests

2. Determine if each function has an inverse. (Sketch a picture)

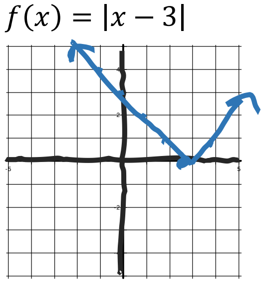
a) $f(x) = x^2 - 9$

f is a function, but not one-to-one

$\boxed{\text{∴ f has no inverse}}$



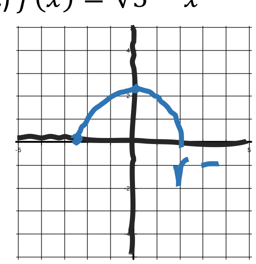
b) $f(x) = |x - 3|$



No inverse

c) $f(x) = \sqrt{5 - x^2}$

$r = \sqrt{5} \approx 2.2$

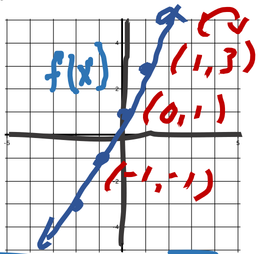


No inverse

4.5 Inverse Functions

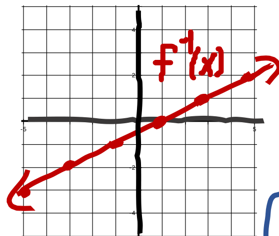
inverse of f

d) $f(x) = 2x + 1$



f has an inverse

e) Given $f(x) = 2x + 1$, find $f^{-1}(x)$



$y = 2x + 1$

$x = 2y + 1$

$\frac{x-1}{2} = y$

$\boxed{f^{-1}(x) = \frac{x-1}{2}}$

$y = \frac{1}{2}x - \frac{1}{2}$

* Switch x and y
* Isolate y
* Substitute g(x) or $f^{-1}(x)$ for y

3. Does the function have an inverse? If so, find a rule (eqtn.) for f^{-1}. Show that they are inverses, then graph.

R: $y \geq 0$ Upper Half

$f(x) = \sqrt{3 - x}$ Yes

$y = \sqrt{3 - x}$

$x = \sqrt{3 - y}$

$x^2 = 3 - y$

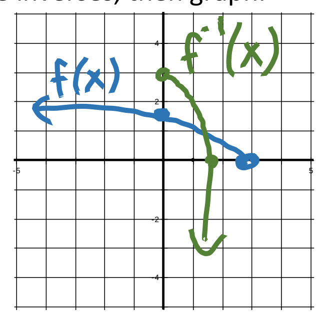
$x^2 - 3 = -y$

$-x^2 + 3 = y$

$\boxed{f^{-1}(x) = -x^2 + 3, \ x \geq 0}$ Right Half of Parabola

D: $x \geq 0$ Right Half

* Switch x and y
* Solve for y
* Substitute $f^{-1}(x)$ for y



$\sqrt{3} \approx 1.732$

4. Find a Rule for $f^{-1}(x)$. Sketch Both.

a) $g(x) = x^2 + 2, x \geq 0$ Domain Right Half

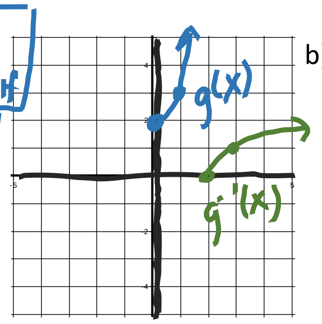
$y = x^2 + 2$

$x = y^2 + 2$

$x - 2 = y^2$

$\cancel{\pm}\sqrt{x - 2} = y$ R: $y \geq 0$ Upper Half

$\boxed{g^{-1}(x) = \sqrt{x - 2}}$



b) $g(x) = 9 - x^2, x \leq 0$ Domain Left Half

$y = 9 - x^2$

$x = 9 - y^2$

$x - 9 = -y^2$

$-x + 9 = y^2$

$\cancel{\pm}\sqrt{-x + 9} = y$ R: $y \leq 0$ Lower Half

$\boxed{g^{-1}(x) = -\sqrt{-x + 9}}$