2.1 Polynomials

HW p. 56 #1 – 9odd,10,11,13,15,21,24-26,29

Polynomial: an expression that has the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ ← n is a whole #

Ex. $x^3 + 2x + 1$, $\frac{5}{8}x$ Non-Ex. $x^{1/2} + 1$, $\frac{1}{x} + 5$

A Polynomial must be **CONTINUOUS:** in other words it must be defined for all values of x.

Ex. (Include at least ONE picture)

NOT Polynomials (Include a picture)

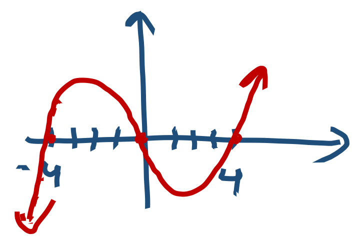
$4x^5 - 2x^3 + 7x^2 - 3$	
	Constant: a number being multiplied to x^0 Ex -3
	Coefficients: a # being multiplied to a variable whose exponent is greater than 0 Ex $4, -2, 7$
	Leading Term: the term whose variable has the highest exponent Ex $4x^5$
	Degree: the highest exponent of the variables Ex 5
	Root: the x-value for which $P(x)=0$; root, solution

#1 a) $P(x) = 2x^3 - 32x$: Find all zeros. $x = ?$ $P(x) = 0$

$0 = 2x^3 - 32x$
$0 = 2x(x^2 - 16)$
$0 = 2x(x-4)(x+4)$
$x = \pm 4, 0$

Polynomial? Yes (cubic)

Rough Sketch:



b) $g(x) = \frac{x+1}{x-1}$ Find all zeros. $x = ?$ $P(x) = 0$

$(x-1)\cdot 0 = \frac{x+1}{\cancel{x-1}}\ \cancel{(x-1)}$

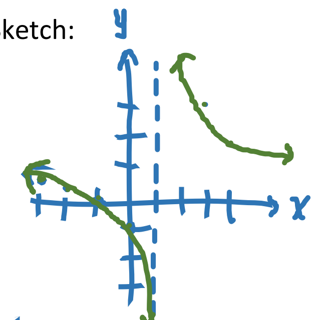
$0 = x + 1$

$\boxed{x = -1}$

Polynomial? No

x	y
-3	+1/2
-2	+1/3
-1	0
0	-1
1	Ø
2	3
4	1 2/3

Rough Sketch:



When is $g(x)$ **undefined?** $x = 1$ Why? denom = 0

Does a value of the zero = the value where the function is undefined? No

Chart of basic Parent Functions						
Parent Function	f(x) = 5	f(x) = x	f(x) = x^2	f(x) = x^3	f(x) = x^4	f(x) = x^5
Degree	0	1	2	3	4	5
Picture						
Name	Constant	Linear	Quadratic	Cubic	Quartic	Quintic

bends = Degree − 1

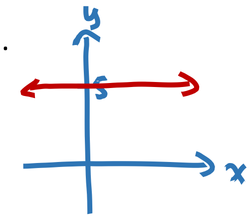
2.1 Polynomials

2\. a) $g(x) = 5$ Find all zeros. $x = ?$ $P(x) = 0$ Polynomial? Sketch.

$0 = 5$

$\boxed{\text{No zeros}}$

Yes (Constant Function)



b) $f(x) = \frac{x^2+4}{2}$ Find all zeros. $x = ?$ $f(x) =$ Polynomial? Yes Sketch.

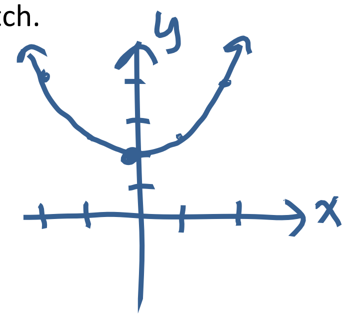
(What is another way to write this function? $f(x) = \frac{1}{2}x^2 + 2$)

$(2)\ 0 = \frac{x^2+4}{2}\ (2)$

$0 = x^2 + 4$

$-4 = x^2$

$\boxed{x = \pm 2i}$



3\. If $P(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10$ $find$:

a) $P(2) = 3(2)^4 - 7(2)^3 - 5(2)^2 + 9(2) + 10$

$\boxed{P(2) = 0}$

b) $P(-3n) = 3(-3n)^4 - 7(-3n)^3 - 5(-3n)^2 + 9(-3n) + 10$

$= 3(243n^4) - 7(-27n^3) - 5(9n^2) - 27n + 10$

$\boxed{P(-3n) = 243n^4 + 189n^3 - 45n^2 - 27n + 10}$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

c) $P(2i) = 3(2i)^4 - 7(2i)^3 - 5(2i)^2 + 9(2i) + 10$

$= 3(16i^4) - 7(8i^3) - 5(4i^2) + 18i + 10$

$= 48(1) - 56(-i) - 20(-1) + 18i + 10$

$= 48 + 56i + 20 + 18i + 10$

$\boxed{= 78 + 74i}$

$x = 2i$ $f(x) = 0$

4\. If **2i is a zero** of $f(x) = x^4 + x^2 + a$, find a.

$\cancel{f(x)}\ 0 = (2i)^4 + (2i)^2 + a$

$0 = 16 - 4 + a$

$0 = 12 + a$

$\boxed{-12 = a}$

5\. A **quadratic** polynomial *P(x)* has **leading coefficient -2,** a **constant term of 6**, but NO **linear term.** Find the zeroes of *P(x)*.

$-2x^2 + 6 = 0$

-6

$\frac{-2x^2}{-2} = \frac{-6}{-2}$

$x^2 = 3$

$\sqrt{x^2} = \sqrt{3}$

$\boxed{x = \pm\sqrt{3}}$

2.1 Polynomials

(Plug-n-Chug)

Direct Substitution Vs. Synthetic Substitution

Find $P(2)$

$P(x) = 2x^3 + 5x^2 + x - 1$

$P(x) = x(2x^2 + 5x + 1) - 1$

$P(2) = 2[2(2(2)+5)+1] - 1$

9

[19]

2	2	5	1	-1
	↓	4	18	38
	2	9	19	37

$\boxed{P(2) = 37}$

6. Find P(-1) for $P(x) = 3x^4 - 7x^3 + 5x^2 - 7$

a) Method 1: ___ict___ substitution.

b) Method 2: ___Synthetic___ substitution.

$P(-1) = 3(-1)^4 + 7(-1)^3 + 5(-1)^2 - 7$

$P(-1) = 3 + 7 + 5 - 7$

$\boxed{P(-1) = 8}$

-1	3	-7	5	0	-7
	↓	-3	10	-15	15
	3	-10	15	-15	8

$\boxed{P(-1) = 8}$

7. Find *S(3)* if $S(x) = 3x^4 - 5x^2 + 9x + 10$. Do this synthetically.

3		0	-5	9	10
	1	9	27	66	225
		9	22	75	235

$\boxed{S(3) = 235}$

8. If $P(x) = 4x^4 - 3x^3 + 7x - 2, find\ P\left(\frac{3}{4}\right)$.

3/4	4	-3	7	-2
	↓	3	0	21/4
	4	0	7	3 1/4

$P(3/4) = \boxed{3\frac{1}{4}}$

9. If $h(x) = 2x^2 - 4x + 3, find\ h(1 + i)$.

$h(1+i) = 2(1+i)^2 - 4(1+i) + 3$

$= 2(1+i)(1+i) - 4 - 4i + 3$

$= 2(1 + 2i + i^2) - 1 - 4i$

$= 2(\cancel{1} + 2i - \cancel{1}) - 1 - 4i$

$= 2(2i) - 1 - 4i$

$= 4i - 1 - 4i$

$\boxed{h(1+i) = -1}$

Why does Synthetic Substitution work? How is it related to Direct Substitution?

P(   ) = Numerical Value or Expression

Find a zero.

$x = ?\ f(x) = 0$