13.6 Sigma Notation

Σ ← sum

Limits of summation (Must be whole #'s)

HW p. 508 #3 – 21odd, 25,27

$1^2 + 2^2 + 3^2 + \ldots + 100^2$

$$\sum_{n=a}^{b} t_n$$

Formula to find each term (summand)

Express in Sigma Notation:

Last # to plug in n

$$\sum_{n=1}^{100} n^2$$

index

1st # to plug in to n

Formula for t_n

Reads: The sum of n^2 for values of n from 1 to 100.

index means the symbol you're using

1. Expand the series.

a) $\sum_{n=1}^{5} 2n$

$= 2(1) + 2(2) + 2(3) + 2(4) + 2(5)$

$\boxed{= 2 + 4 + 6 + 8 + 10}$

b) $\sum_{k=1}^{\infty} \frac{1}{k}$

$= \frac{1}{(1)} + \frac{1}{(2)} + \frac{1}{(3)} + \ldots + \frac{1}{k} + \ldots$

$\boxed{= 1 + \frac{1}{2} + \frac{1}{3} + \ldots}$

2. Write using sigma notation.

a) $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4$

$\boxed{= \sum_{n=1}^{4} \left(\frac{1}{2}\right)^n}$

b) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots$ Neither

$t_1 \; t_2 \; t_3 \; t_4$

$\boxed{\sum_{n=1}^{\infty} \frac{n}{n+1}}$

c) 4 + 7 + 10 + ... + 67 Arithmetic

$\Sigma t_n \rightarrow \boxed{\sum_{n=1}^{22} 3n+1}$

$t_n = t_1 + (n-1)d$

$t_n = 4 + (n-1)(3)$

$t_n = 3n + 1$

$67 = 4 + (n-1)(3)$

$67 = 3n + 1$

$66 = 3n$

$n = 22$

d) Expand and Evaluate: $\sum_{n=2}^{5} \log \sqrt[n]{10}$

$= \log\sqrt{10} + \log\sqrt[3]{10} + \log\sqrt[4]{10} + \log\sqrt[5]{10}$

$= \log 10^{1/2} + \log 10^{1/3} + \log 10^{1/4} + \log 10^{1/5}$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

$\boxed{= \frac{77}{60}}$

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$t_1 \quad t_2 \quad t_3$

e) -2 + 4 – 6 + … + 100

$$\boxed{\sum_{n=1}^{50} 2n(-1)^n}$$

or

$t_n = t_1 + (n-1)d$

$t_n = 2 + (n-1)(2)$

$t_n = 2n$

alternating signs

Neither

or

use Arithmetic with alternating signs

f) 1 + 2 + 4 + 8 + 16 + 32

Geometric

$$\boxed{\sum_{n=1}^{6} (2)^{n-1}} \quad \text{or} \quad \boxed{\sum_{n=0}^{5} 2^n}$$

$t_n = t_1 \cdot r^{n-1}$

$t_n = 1 \cdot (2)^{n-1}$

3. Write the series in expanded form.

a) $\sum_{n=2}^{6} \frac{1}{2n}$

$= \frac{1}{2(2)} + \frac{1}{2(3)} + \frac{1}{2(4)} + \frac{1}{2(5)} + \frac{1}{2(6)}$

$\boxed{= \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12}}$

b) $\sum_{n=3}^{\infty} n(-1)^n$

$= 3(-1)^3 + 4(-1)^4 + 5(-1)^5 + \ldots$

$\boxed{= -3 + 4 - 5 + \cdots}$

4. Show that $\sum_{n=2}^{5} \log n = \cancel{120}$ log120

$= \log 2 + \log 3 + \log 4 + \log 5$

$= \log(2 \cdot 3 \cdot 4 \cdot 5)$ ✓

$\boxed{= \log 120}$

5.) Express in Σ notation

$24 - 12 + 6 - 3 + \frac{3}{2}$

$\times -\frac{1}{2}$ $\quad t_n = 24 \cdot \left(-\frac{1}{2}\right)^{n-1}$

Geo

$$\boxed{\sum_{n=1}^{5} 24\left(-\tfrac{1}{2}\right)^{n-1} \quad \text{or} \quad \sum_{n=0}^{4} 24\left(-\tfrac{1}{2}\right)^{n}}$$

6.) Express in Σ notation: $2 - 5 + 8 - 11 + \cdots - 50$

$t_n = 2 + (n-1)(3)$

$t_n = 3n - 1$

$$\boxed{\sum_{n=1}^{17} (3n-1)(-1)^{n-1}}$$

$50 = 2 + (n-1)(3)$

$50 = 3n - 1$

$51 = 3n$

$n = 17$