11.4 Roots of a Complex Number

HW p. 413 #2,4,6,7,12-15,17

1. Solve $\sqrt[4]{x^4} = \sqrt[4]{16}$

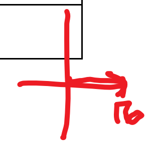
$\boxed{x = 2, -2, 2i, -2i}$

Check: $(2i)^4 = 2^4 i^4$
$= 16(1)$ ✓

$\boxed{\begin{array}{l} 2\operatorname{cis}0^\circ \\ 2\operatorname{cis}90^\circ \\ 2\operatorname{cis}180^\circ \\ 2\operatorname{cis}270^\circ \end{array}}$

2. Find the fourth roots of 16.

$16+0i$



Convert to polar form:

$r = \sqrt{(16)^2+(0)^2}$ $\quad \tan\theta = \frac{0}{16}$

$r = 16$ $\quad \theta = 0^\circ$

$= 16\operatorname{cis}0^\circ$

Solve $z^4 = 16$

$z^4 = 16\operatorname{cis}0^\circ$

$(r\operatorname{cis}\theta)^4 = 16\operatorname{cis}0^\circ$

$r^4\operatorname{cis}4\theta = 16\operatorname{cis}0^\circ$

$\sqrt[4]{r^4} = \sqrt[4]{16}$

$\boxed{r = 2}$

$\frac{4\theta}{4} = \frac{0^\circ + 360n}{4}$

$\frac{4\theta}{4} = \frac{0^\circ + 360(0)}{4}$

$\frac{4\theta}{4} = \frac{0^\circ + 360(1)}{4}$

$\frac{4\theta}{4} = \frac{0^\circ + 360(2)}{4}$

$\frac{4\theta}{4} = \frac{0^\circ + 360(3)}{4}$

$\boxed{\begin{array}{l} \theta_1 = 0^\circ \\ \theta_2 = 90^\circ \\ \theta_3 = 180^\circ \\ \theta_4 = 270^\circ \end{array}}$

nth Roots of a Complex Number in Polar Form:

$$\sqrt[n]{z} = \sqrt[n]{r}\operatorname{cis}\left(\frac{\theta + 360^\circ \cdot k}{n}\right) \qquad k = 0, 1, 2, \ldots, n-1$$

3. Find the cube roots of 8i.

$0+8i$

$r = \sqrt{8^2+0^2}$ $\quad \tan\theta = \frac{8}{0}$

$r = 8$ $\quad \theta = 90^\circ$

$z = 8\operatorname{cis}90^\circ$

$\sqrt[3]{z} = \sqrt[3]{8}\operatorname{cis}\left(\frac{90^\circ + 360^\circ(0)}{3}\right) = \boxed{2\operatorname{cis}30^\circ}$

$= \sqrt[3]{8}\operatorname{cis}\left(\frac{90^\circ + 360^\circ(1)}{3}\right) = \boxed{2\operatorname{cis}150^\circ}$

$= \sqrt[3]{8}\operatorname{cis}\left(\frac{90^\circ + 360^\circ(2)}{3}\right) = \boxed{2\operatorname{cis}270^\circ}$

4. Find the fourth roots of $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Q3

$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2}$

$r = \sqrt{\frac{1}{4} + \frac{3}{4}}$

$r = 1$

$\tan\theta = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

ref∠ $= 60^\circ$

Q3: 240°

$1\operatorname{cis}240^\circ$

$\sqrt[4]{1}\operatorname{cis}\left(\frac{240^\circ + 360^\circ(0)}{4}\right)$

$\sqrt[4]{1}\operatorname{cis}\left(\frac{240^\circ + 360^\circ(1)}{4}\right)$

$\sqrt[4]{1}\operatorname{cis}\left(\frac{240^\circ + 360^\circ(2)}{4}\right)$

$\sqrt[4]{1}\operatorname{cis}\left(\frac{240^\circ + 360^\circ(3)}{4}\right)$

$\boxed{\begin{array}{l} = 1\operatorname{cis}60^\circ \\ = 1\operatorname{cis}150^\circ \\ = 1\operatorname{cis}240^\circ \\ = 1\operatorname{cis}330^\circ \end{array}}$

11.4 Roots of a Complex Number

HW p. 413 #2,4,6,7,12-15,17

5. The 3 cube roots of 8 must satisfy the equation $z^3 - 8 = 0$. Solve this equation.

$$z^3 = 8$$

$$\sqrt[3]{z^3} = \sqrt[3]{8}$$

Find cube roots of $8 + 0i$

$r = \sqrt{(8)^2 + (0)^2}$

$r = 8$

$\tan\theta = \frac{0}{8}$

$\theta = 0°$

$8 \text{ cis } 0°$

$$\sqrt[3]{8} = \sqrt[3]{8} \text{ cis}\left(\frac{0° + 360°(0)}{3}\right) = \boxed{2 \text{ cis } 0°}$$

$$\sqrt[3]{8} \text{ cis}\left(\frac{0° + 360°(1)}{3}\right) = \boxed{2 \text{ cis } 120°}$$

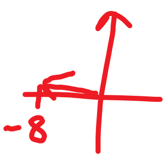
$$\sqrt[3]{8} \text{ cis}\left(\frac{0° + 360°(2)}{3}\right) = \boxed{2 \text{ cis } 240°}$$

6. Solve $z^3 = -8$

$$\sqrt[3]{z^3} = \sqrt[3]{-8}$$

$$z = \sqrt[3]{-8}$$

$8 \text{ cis } 180°$



-8

$r = 8$

$\theta = 180°$

$$\sqrt[3]{-8} = \sqrt[3]{8} \text{ cis}\left(\frac{180° + 360°(0)}{3}\right)$$

$$= \sqrt[3]{8} \text{ cis}\left(\frac{180° + 360°(1)}{3}\right)$$

$$= \sqrt[3]{8} \text{ cis}\left(\frac{180° + 360°(2)}{3}\right)$$

$$\boxed{\begin{aligned} &= 2 \text{ cis } 60° \\ &= 2 \text{ cis } 180° \\ &= 2 \text{ cis } 300° \end{aligned}}$$

7. a) Using Example #2 from p.413, show that
$z_1 + z_2 + z_3 + z_4 = 0$ *and* $z_1 z_2 z_3 z_4 = 16$

$$z_1 + z_2 + z_3 + z_4 = (\sqrt{2} + i\sqrt{2}) + (-\sqrt{2} + i\sqrt{2}) + (-\sqrt{2} - i\sqrt{2}) + (\sqrt{2} - i\sqrt{2})$$

$$0 + 0 \checkmark$$

$$z_1 z_2 z_3 z_4 = (2 \cdot 2 \cdot 2 \cdot 2) \text{ cis}(45° + 135° + 225° + 315°)$$

$$= 16 \text{ cis } 720°$$

$$= 16 \text{ cis } 0°$$

or $\boxed{16}$

b) Any fourth root of -16 must satisfy $z^4 = -16$.
Show that part (a) verifies Theorem 5 on p.86

$$x^4 = -16$$

$$x^4 + 16 = 0$$

$$\overset{a_n}{x^4} + \overset{a_{n-1}}{0x^3} + 0x^2 + 0x + \overset{a_0}{16} = 0$$

Sum of roots: $-\frac{a_{n-1}}{a_0} = -\frac{0}{1} = \boxed{0} \checkmark$

product of roots: $\frac{a_0}{a_n} = \frac{16}{1} = \boxed{16} \checkmark$