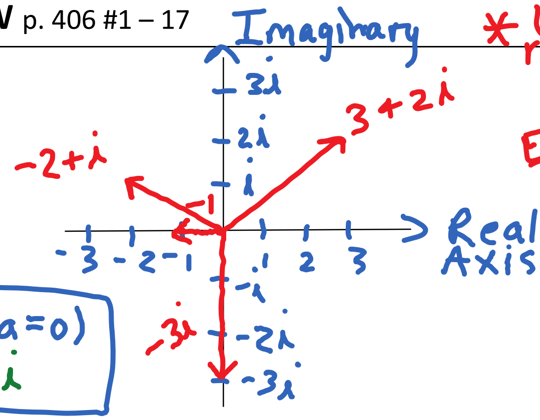
11.2 Complex Number Plane

HW p. 406 #1 – 17

* Use an arrow to represent complex #'s



Ex Graph
a) $3+2i$
b) -1
c) $-3i$
d) $-2+i$

The Complex Plane (Aragand Diagram)

Complex #'s $\mathbb{C}$

$a + bi$
real imaginary

Ex. $1,\ 3i,\ 4+2i$

Real $(b=0)$
$1, -\frac{1}{2}, 0, -4, 1.2, \pi, \sqrt{2}$

Imaginary $(a=0)$
$3i, -i, -\sqrt{10}\,i$

Complex Coordinate (z) in **Polar Form** vs. Complex Coordinate (z) in **Rectangular Form**

$z = a + bi$ (x, y)

$z = r\cos\theta + i r\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

$z = r\,cis\,\theta$

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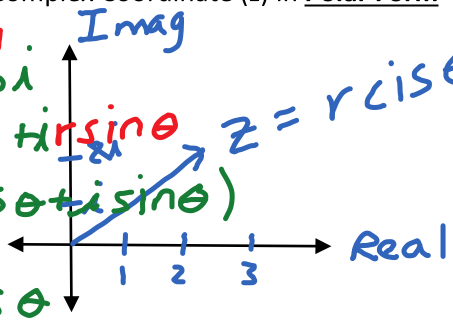
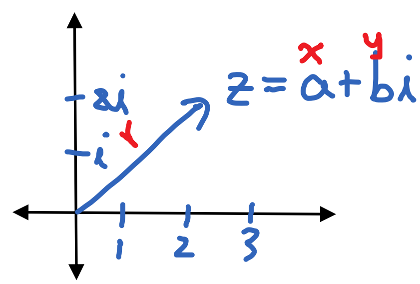


$z = a + bi$ (x, y)

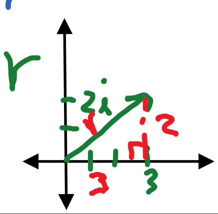


Absolute Value of a Complex Number: $|z| = |a + bi|$: Find the length of the arrow (r)

- The length of an arrow representing a complex number. If $z = a + bi$, then $|z| = r$

- Length of Arrow: r = $|a+bi| = \sqrt{a^2+b^2}$ $|r\,cis\,\theta| = r$
- Example: Find $|3 + 2i| =$ $r = \sqrt{(3)^2+(2)^2} = \sqrt{13}$

$|2\,cis\,30^\circ| = 2$



COMPLEX COORDINATES

Polar → Rectangular: Given $r\,cis\,\theta$ → $a+bi$

$a = r\cos\theta$ $b = r\sin\theta$

$\boxed{a+bi}$ (x, y)

Rectangular → Polar: Given $a + bi$ (x, y) → $r\,cis\,\theta$

$r^2 = a^2 + b^2$ $\tan\theta = \frac{b}{a}$

$\boxed{r\,cis\,\theta}$

1. Express each complex number in polar form: $r\,cis\,\theta$

a) $-5 + i$ Q2

$r^2 = a^2 + b^2$

$r = \sqrt{(-5)^2 + (1)^2}$

$r = \sqrt{26}$

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{1}{-5}$

ref∠ ≈ 11.3°

Q2: 168.7°

$\boxed{\sqrt{26}\,cis\,168.7^\circ}$

b) $2\sqrt{3} + 2i$ Q1

$r^2 = a^2 + b^2$

$r = \sqrt{(2\sqrt{3})^2 + (2)^2}$

$r = \sqrt{12+4}$

$r = 4$

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{2}{2\sqrt{3}}$

ref∠ = 30°

Q1: 30°

$\boxed{4\,cis\,30^\circ}$

2. Express each complex number in rectangular form: $a + bi$

a) $3\ cis\ 100^o$ Q2

$a = 3\cos 100^\circ$ $\quad b = 3\sin 100^\circ$

$a \approx -0.52$ $\quad b \approx 2.95$

$\boxed{-0.52 + 2.95i}$

b) $8\ cis\ \frac{7\pi}{6}$ Q3

$a = 8\cos\frac{7\pi}{6}$ $\quad b = 8\sin\frac{7\pi}{6}$

$a = 8\left(\frac{\sqrt{3}}{2}\right)$ $\quad b = 8\left(\frac{1}{2}\right)$

$a = 4\sqrt{3}$ $\quad b = 4$

$\boxed{4\sqrt{3} + 4i}$

Proof: Multiplying Complex #'s (Polar Form)

Find $r_1 cis \propto \cdot r_2 cis\beta =$

$r_1(\cos\alpha + i\sin\alpha)\cdot r_2(\cos\beta + i\sin\beta)$

$= (r_1\cos\alpha + r_1 i\sin\alpha)\cdot(r_2\cos\beta + r_2 i\sin\beta)$

$= r_1r_2\cos\alpha\cos\beta + r_1r_2 i\cos\alpha\sin\beta + r_1r_2 i\sin\alpha\cos\beta + r_1r_2 i^2\sin\alpha\sin\beta$

$= r_1r_2(\cos\alpha\cos\beta - \sin\alpha\sin\beta) + r_1r_2 i(\cos\alpha\sin\beta + \sin\alpha\cos\beta) = r_1r_2\cos(\alpha+\beta) + r_1r_2 i\sin(\alpha+\beta)$

$= r_1r_2(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$

$\boxed{= r_1r_2\ cis(\alpha+\beta)}$

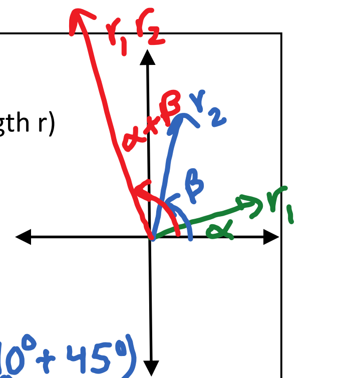
Multiplying 2 Complex #'s (Polar Form)

1) Multiply their absolute values (length r)

2) Add their polar angles

$z_1z_2 = r_1r_2\ cis(\alpha+\beta)$

Ex. $(2\ cis\ 10^o)(4\ cis\ 45^o) = 2\cdot 4\ cis(10^\circ + 45^\circ)$
$= 8\ cis\ 55^\circ$



3. Give the product in Polar + Rectangular form:

a) $(3\ cis\ 165^o)(4\ cis\ 45^o) = 3\cdot 4\ cis(165^\circ + 45^\circ)$

$\boxed{= 12\ cis\ 210^\circ}$

$a = 12\cos 21$ $\quad b = 12\sin 210^\circ$

$a = 12\left(-\frac{\sqrt{3}}{2}\right)$ $\quad b = 12\left(-\frac{1}{2}\right)$

$-\sqrt{3} - 6$

4. $z_1 = -3 - 5i, z_2 = 4 + 7i$

a) Find z_1z_2 in rectangular form (a + bi)

$= (-3 - 5i)(4 + 7i)$

$= -12 - 21i - 20i - 35i^2$

$-12 - 41i - 35(-1)$

$\boxed{= 23 - 41i}$

b) Find z_1, z_2 and z_1z_2 in polar form ($r\ cis\ \theta$).

$z_1 = -3 - 5i$: $r = \sqrt{(-3)^2 + (-5)^2}$ $\quad \tan\theta = \frac{-5}{-3}$

$r = \sqrt{34}$ $\quad$ ref∠ $\approx 59^\circ$

$\sqrt{34}\ cis\ 239^\circ$ $\quad$ Q3: 239°

$z_2 = 4 + 7i$: $r = \sqrt{4^2 + 7^2}$ $\quad \tan\theta = \frac{7}{4}$

$r = \sqrt{65}$ $\quad$ ref∠ $\approx 60.3^\circ$

$\sqrt{65}\ cis\ 60.3^\circ$ $\quad$ QI

$z_1z_2 = \sqrt{34}\cdot\sqrt{65}\ cis(239^\circ + 60.3^\circ) = \boxed{\sqrt{2210}\ cis\ 299.3^\circ}$

c) Show that z_1z_2 in part b agrees with the answer in part a.

z_1z_2: $23 - 41i$

$r^2 = a^2 + b^2$ $\quad \tan\theta = \frac{b}{a}$

$r = \sqrt{(23)^2 + (-41)^2}$ $\quad \tan\theta = -\frac{41}{23}$

$r = \sqrt{2210}$ $\quad$ ref∠ $\approx 60.7^\circ$

Q4: 299.3°

✓ $\boxed{\sqrt{2210}\ cis\ 299.3^\circ}$

d) Show z_1, z_2 and z_1z_2, in an Argand Diagram