# 11.1 Polar Coordinates

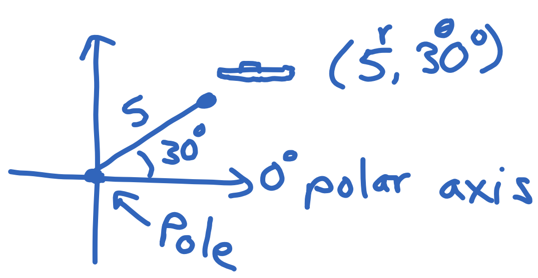
**HW** p. 400 #1 – 15odd, 25

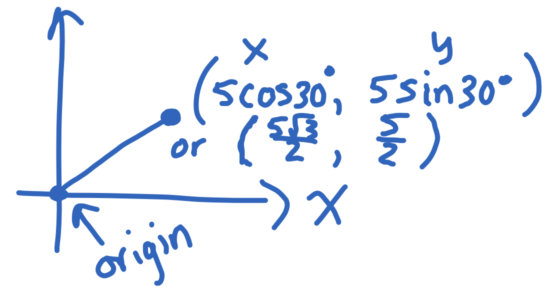
$(r, \theta)$ Polar Coordinates vs. Rectangular Coordinates $(x, y)$

dist dir.

-Based off <u>distance</u> and <u>angle</u> (r, θ)

- Based off <u>x</u> and <u>y</u>.

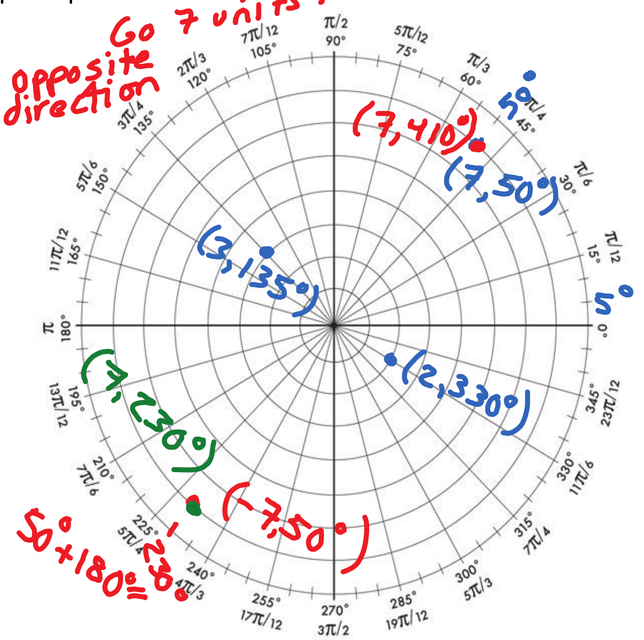


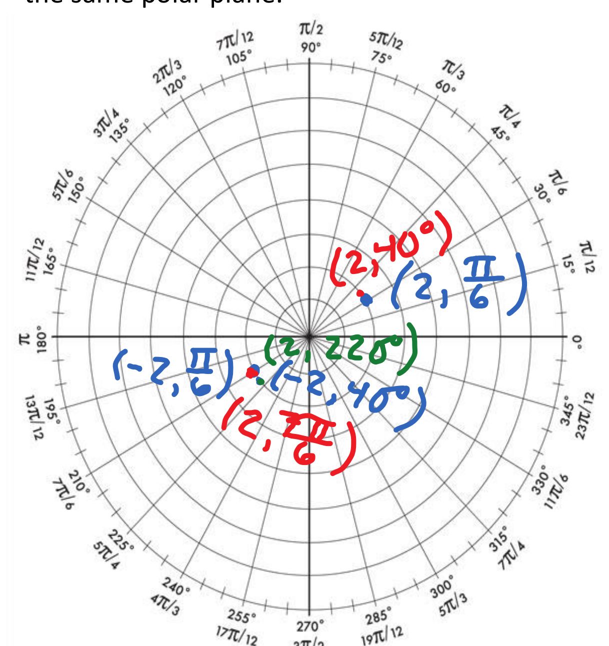

1. Sketch the Polar Coordinates $(3, 135^o)$, $(2, 330^0)$, $(7, 50^o)$, $(7, 410^o)$, $(-7, 50^o)$, $(7, 230^o)$ on the same polar plane.

ring

Go 7 units in opposite direction




2. Sketch the Polar Coordinates $\left(2, \frac{\pi}{6}\right), \left(-2, \frac{\pi}{6}\right), \left(2, \frac{7\pi}{6}\right)$ $(2, 40^o)$, $(-2, 40^0)$, $(2, 220^o)$n the same polar plane.




Q3 $(-2, 40°)$ is the same location as Q3 $(2, 220°)$, Q3 $(-2, 400°)$, Q3 $(2, 580°)$, Q3 $(2, -140°)$

* "r" is <u>negative</u> when you go the same <u>#</u> of units in the <u>opposite</u> direction

(Add or subtract 180°)

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3. Give 2 other pairs of polar coordinates for given point. (*FLIP opp r sign, add or subtract 180°)

* Add or sub. 360° – don't change sign of r

a) $(5, 60^o)$

$(-5, 60^\circ + 180^\circ) = \boxed{(-5, 240^\circ)}$

$(5, 60^\circ + 360^\circ) = \boxed{(5, 420^\circ)}$

b) $(-3, 110^o)$

$(+3, 110^\circ + 180^\circ) = \boxed{(+3, 290^\circ)}$

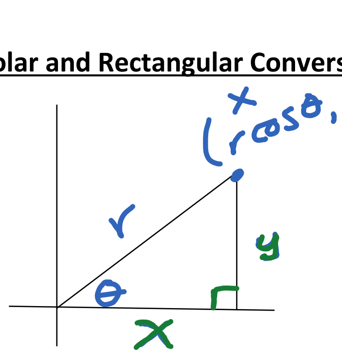
$(-3, 110^\circ + 360^\circ) = \boxed{(-3, 470^\circ)}$

c) $\left(4, \frac{\pi}{2}\right)$

$\left(4, \frac{\pi}{2} + \pi\right) = \boxed{\left(-4, \frac{3\pi}{2}\right)}$

$\left(4, \frac{\pi}{2} + 2\pi\right) = \boxed{\left(4, \frac{5\pi}{2}\right)}$

**Polar and Rectangular Conversions**




To go from **POLAR** ⟶ **Rectangular:** Given $(r, \theta)$

$x = r\cos\theta \qquad y = r\sin\theta$

$(x, y)$

To go from **Rectangular** ⟶ **Polar:** Given (x,y)

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

* don't use neg. r

$\tan\theta = \frac{y}{x}$

↑ Quad. (x, y) is located in

$(r, \theta)$

5. Give polar coordinates $(r, \theta)$ for each point.

a) $(-4, -4)$, $\theta$ in degrees.

Q3

$r^2 = x^2 + y^2$

$r = \sqrt{(-4)^2 + (-4)^2}$

$r = \sqrt{16 + 16}$

$r = \sqrt{32}$

$r = 4\sqrt{2}$ $\quad \boxed{(4\sqrt{2}, 225^\circ)}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-4}{-4}$

$\tan\theta = 1$

ref ∠ = 45°

Q3: 225°

b) $(-\sqrt{3}, 3)$, $\theta$ in radians

6. Give rectangular coordinates $(x, y)$ for each point (EXACT when possible).

a) $(6, -30^o)$

Q4

$x = r\cos\theta$

$x = 6\cos(-30^\circ)$

$x = 6\left(\frac{\sqrt{3}}{2}\right)$

$x = 3\sqrt{3}$

$y = r\sin\theta$

$y = 6\sin(-30^\circ)$

$y = 6\left(-\frac{1}{2}\right)$

$y = -3$

$\boxed{(3\sqrt{3}, -3)}$

b) $(2, 3)$

Q2 since $\pi \approx 3.14$

$x = r\cos\theta$

$x = 2\cos 3$

$x \approx -1.98$

$y = r\sin\theta$

$y = 2\sin 3$

$y \approx 0.28$

$\boxed{(-1.98, 0.28)}$

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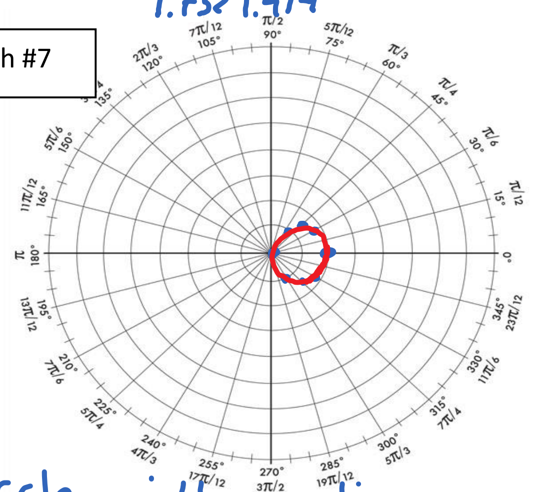
7. Sketch $r = 2\cos\theta$

| $\theta$ | | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r | | $\sqrt{3}$ | $\sqrt{2}$ | 1 | 0 | $-1$ | $-\sqrt{2}$ | $-\sqrt{3}$ | $-2$ | $-\sqrt{3}$ | $-\sqrt{2}$ | $-1$ | 0 | 1 | $\sqrt{2}$ | $\sqrt{3}$ | 2 |

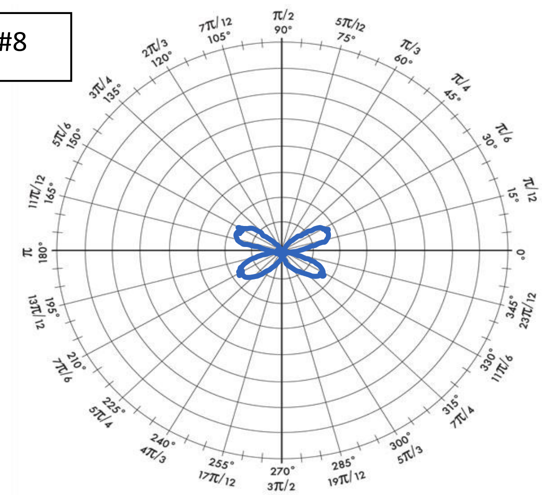
1.732 1.414

Graph #7



* circle with radius 1 centered on the x-axis

Graph #8



* A flower with 4 petals

8. Try $r = 2\sin 2\theta$

| $\theta$ | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r | 0 | $\sqrt{3}$ | 2 | $\sqrt{3}$ | 0 | $\sqrt{3}$ | 2 | $\sqrt{3}$ | 0 | $-\sqrt{3}$ | $-2$ | $-\sqrt{3}$ | 0 | $-\sqrt{3}$ | $-2$ | $-\sqrt{3}$ | 0 |

9. Find a rectangular equation: (Here's the GOAL: x and y / r and θ in terms of x and y with NO fractional exponents).

$$r^2 = x^2 + y^2 \qquad \sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

a) $r = 2\sin 2\theta$

$$r = 2(2\sin\theta\cos\theta)$$
$$r = 4\sin\theta\cos\theta$$
$$r = 4\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$$
$$r = \frac{4xy}{r^2}$$
$$\left(\pm\sqrt{x^2+y^2}\right)^2 = \left(\frac{4xy}{x^2+y^2}\right)^2$$
$$\boxed{x^2 + y^2 = \left(\frac{4xy}{x^2+y^2}\right)^2}$$

b) $r = \cos\theta$ Circle

$$r = \left(\frac{x}{r}\right)$$

* Mult. by r

$$r^2 = x$$
$$x^2 + y^2 = x$$
$$x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4}$$

$*\left(\frac{b}{2}\right)^2$ $\left(\frac{-1}{2}\right)^2$

$$\boxed{\left(x - \tfrac{1}{2}\right)^2 + y^2 = \frac{1}{4}}$$

Center $\left(\frac{1}{2}, 0\right)$ $r = \frac{1}{2}$

c) $r = 1 + \cos\theta$

$$r \cdot r = \left[1 + \left(\frac{x}{r}\right)\right] \cdot r$$
$$r^2 = r + x$$
$$x^2 + y^2 = \pm\sqrt{x^2+y^2} + x$$
$$x^2 + y^2 - x = \pm\sqrt{x^2+y^2}$$
$$\boxed{(x^2 + y^2 - x)^2 = x^2 + y^2}$$

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POLAR BEAR

CARTESIAN BEAR

## Special Polar Graphs

Several important types of graphs have equations that are simpler in polar form than in rectangular form. For example, the circle

$$r = 4\sin\theta$$

in Example 1 has the more complicated rectangular equation

$$x^2 + (y-2)^2 = 4.$$

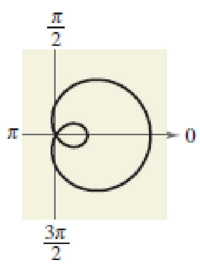
Several other types of graphs that have simple polar equations are shown below.

*Limaçons*

$r = a \pm b\cos\theta$

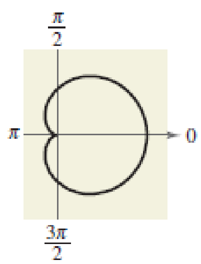
$r = a \pm b\sin\theta$

$(a > 0, b > 0)$



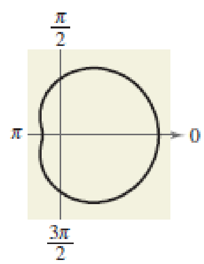

$\frac{a}{b} < 1$

Limaçon with inner loop



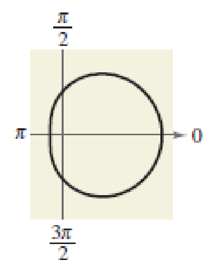

$\frac{a}{b} = 1$

Cardioid (heart-shaped)




$1 < \frac{a}{b} < 2$

Dimpled limaçon

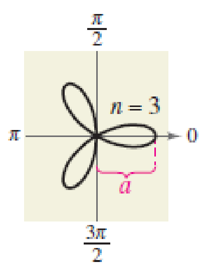



$\frac{a}{b} \geq 2$

Convex limaçon

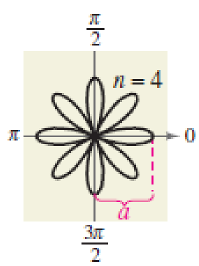
*Rose Curves*

$n$ petals if $n$ is odd,
$2n$ petals if $n$ is even
$(n \geq 2)$



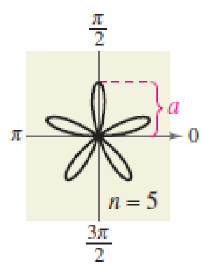

$r = a\cos n\theta$

Rose curve



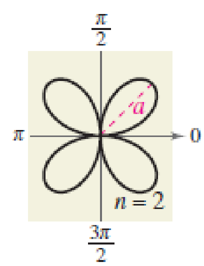

$r = a\cos n\theta$

Rose curve




$r = a\sin n\theta$

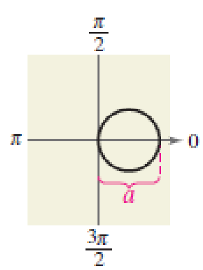
Rose curve




$r = a\sin n\theta$

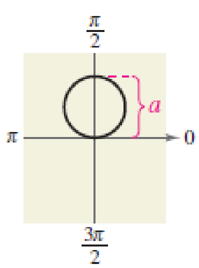
Rose curve

*Circles and Lemniscates*



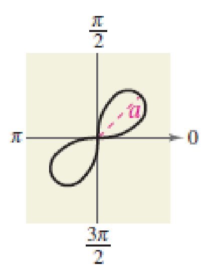

$r = a\cos\theta$

Circle



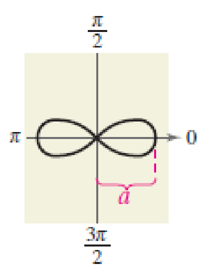

$r = a\sin\theta$

Circle




$r^2 = a^2\sin 2\theta$

Lemniscate




$r^2 = a^2\cos 2\theta$

Lemniscate

Spiral of Archimedes: $r = a\theta$  Hyperbolic spiral: $r = \dfrac{1}{\theta}$