* Get on one sides factor out GCF
* Make a tangent $\sin x = \cos x$
* Factoring
* Distribute
* common denom.
* Trig. Identity

10.4 Solving Trigonometric Equations

HW pg. 389 #11, 12, 15, 17 – 32

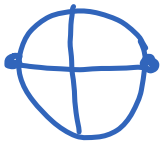
1. Solve $2\sin x\cos x = \sin x \; for \; 0^o \le x < 360^o$

*Get one one side (GCF)

$2\sin x\cos x - \sin x = 0$

$\sin x \cdot (2\cos x - 1) = 0$

$\sin x = 0$



$x = 0°, 180°$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

QI: 60°
Q4: 300°

2. Solve $2\sin(x + 30^o) = 1 \; for \; 0^o \le x < 360^o$

Sin⁻¹ $\sin(x+30°) = \frac{1}{2}$ Sin⁻¹

$x + 30° = 30°, 150°$
QI, Q2

$x = 0°, 120°$

*Sub. 30°

3. $\frac{\tan x}{1-\tan^2 x} = \frac{1}{2} \; for \; 0^o \le x < 360^o$

*Mult by 2

$\frac{2\tan x}{1-\tan^2 x} = 1$

Tan⁻¹ $\tan 2x =$ Tan⁻¹ 1

Look between 0° and 2·360° since tangent goes through 2 cycles before 360°

$2x = \underset{Q1}{45°}, \underset{Q3}{225°}, \underset{45°+360°}{405°}, \underset{225°+360°}{585°}$

*Div. by 2 shrinking graph

$x = 22.5°, 112.5°, 202.5°, 292.5°$

4. $\tan 2x = \tan x \; for \; 0 \le x < 2\pi$

*Mult by $(1-\tan^2 x)$

$\frac{2\tan x}{1-\tan^2 x} = \tan x(1-\tan^2 x)$

$2\tan x = \tan x(1-\tan^2 x)$

$2\tan x = \tan x - \tan^3 x$

$0 = -\tan x - \tan^3 x$

$0 = -\tan x(1+\tan^2 x)$

$\tan x = 0$

$x = \underset{\pi}{0}$

$1 + \tan^2 x = 0$

$\sqrt{\tan^2 x} = \sqrt{-1}$

Impossible

5. $\cos 2x = 8\sin^2 x - 2\cos^2 x \; for \; 0 \le x < 2\pi$

$\cos^2 x - \sin^2 x = 8\sin^2 x - 2\cos^2 x$

*make tan

$0 = 9\sin^2 x - 3\cos^2 x$

$\frac{3\cos^2 x}{\cos^2 x} = \frac{9\sin^2 x}{\cos^2 x}$

$3 = 9\tan^2 x$

$\sqrt{\frac{1}{3}} = \sqrt{\tan^2 x}$

Tan⁻¹ $\pm\frac{1}{\sqrt{3}} =$ Tan⁻¹ $\tan x$

$30°, 150°, 210°, 330° = x$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} = x$

6. $2\sin(30^o + x) = 3\cos x \; for \; 0^o \le x < 360^o$

$\sin(30° + x) = \frac{3}{2}\cos x$

$\sin 30°\cos x + \cos 30°\sin x = \frac{3}{2}\cos x$

$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{3}{2}\cos x$

$\frac{\frac{\sqrt{3}}{2}\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\frac{\sqrt{3}}{2}\tan x = 1$

Tan⁻¹ $\tan x =$ Tan⁻¹ $\frac{2}{\sqrt{3}}$

$x = \underset{Q1}{49.1°}, \underset{Q3}{229.5°}$

7. $\cos x = \cos 2x$ *for* $0^o \le x < 360^o$

$\cos x = 2\cos^2 x - 1$

$0 = 2\cos^2 x - \cos x - 1$

$0 = (2\cos x + 1)(\cos x - 1)$

$2\cos x + 1 = 0$ $\quad \cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\quad \cos x = 1$

$\cos^{-1}$ $\quad$ QR: $\frac{1}{2}$

$\text{ref}\angle = 60°$

$x = 120°, 240°$ (Q2, Q3) $\quad x = 0°$

8. $\sin x \cos 2x = 1$ *for* $0 \le x < 2\pi$

$\sin x(1 - 2\sin^2 x) = 1$

$\sin x - 2\sin^3 x = 1$

$0 = 2\sin^3 x - \sin x + 1$

Rational Root Thm.

$x = \frac{p}{q} = \frac{\pm 1}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}$

	2	0	−1	1
−1		−2	2	−1
	$2x^2$	$-2x$	$+1$	0

$(x+1)(2x^2 - 2x + 1) = 0$

$(\sin x + 1)(2\sin^2 x - 2\sin x + 1) = 0$

$\sin x = -1$ $\quad x = \frac{3\pi}{2}$

$\sin x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(1)}}{2(2)}$

$\sin x = \frac{2 \pm \sqrt{-4}}{4}$

9. Determine where $y = \sin x$ intersects $y = \sin 2x$ over $0^0 \le x < 360^o$.

$\sin x = \sin 2x$

$\sin x = 2\sin x \cos x$

$0 = 2\sin x \cos x - \sin x$

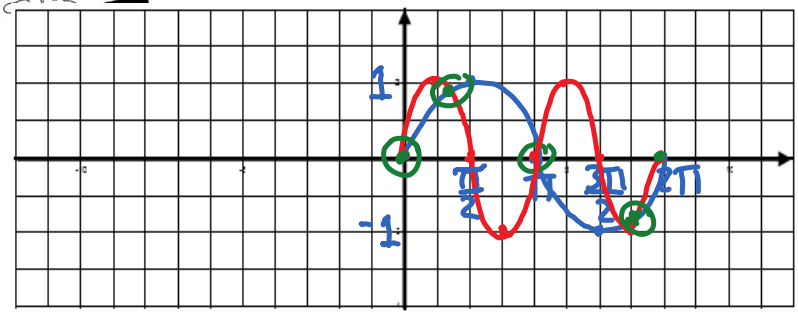
$0 = \sin x(2\cos x - 1)$

$\sin x = 0$ $\quad 2\cos x - 1 = 0$

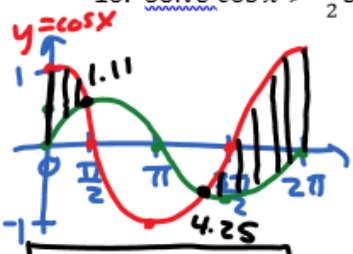
$\cos^{-1}$ $\quad \cos x = \frac{1}{2}$

$x = 0°, 180°$ $\quad \text{ref}\angle = 60°$

$x = 60°, 300°$ (Q1, Q4)



10. Solve $\cos x > \frac{1}{2}\sin x$ *for* $0 \le x < 2\pi$.



$\frac{\cos x}{\cos x} = \frac{\frac{1}{2}\sin x}{\cos x}$

$1 = \frac{1}{2}\tan x$

$2 = \tan x$

$\text{ref}\angle \approx 1.11$

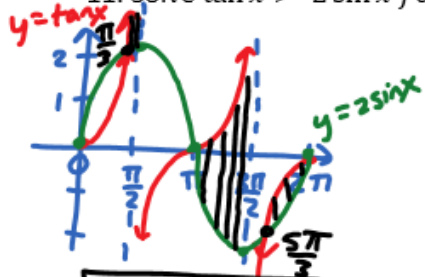
QI: 1.11

QIII: $\pi + 1.11 = 4.25$

$0 \le x < 1.11$

$4.25 < x \le 2\pi$

11. Solve $\tan x > 2\sin x$ *for* $0 \le x < 2\pi$



$\tan x = 2\sin x$

$\frac{\sin x}{\cos x} = 2\sin x$

$\sin x = 2\sin x \cos x$

$0 = 2\sin x \cos x - \sin x$

$0 = \sin x(2\cos x - 1)$

$\sin x = 0$ $\quad \cos x = \frac{1}{2}$

$x = 0, \pi$ $\quad \text{ref}\angle = \frac{\pi}{3}$ (60°)

QI: $\frac{\pi}{3}$

QIV: $\frac{5\pi}{3}$

$\frac{\pi}{3} < x < \frac{\pi}{2}$

$\pi < x < \frac{3\pi}{2}$

$\frac{5\pi}{3} < x < 2\pi$