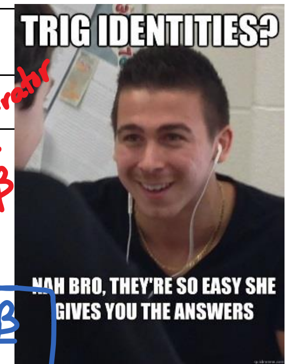
10.2 Formulas for $\tan(\alpha \pm \beta)$ and $\sin(\alpha \pm \beta)$

HW pg. 377 #'s 1 – 8, 11 – 14, 23 – 26



Proof: Find $\tan(\alpha+\beta) = \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$ * Div. numerator & denom. by $\cos\alpha\cos\beta$

$$= \frac{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} = \boxed{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$$

Now find $\tan(\alpha-\beta) = \dfrac{\tan\alpha + \tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$; $\tan(-\beta) = -\tan\beta$; $\dfrac{\tan\alpha - \tan\beta}{\alpha\tan\beta}$

Sum + Difference Formulas for Tangent:

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

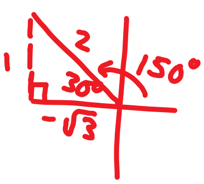
$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

1. Find $\tan(\alpha+\beta)$ given that $\tan\alpha = 2$ and $\tan\beta = -\frac{1}{3}$

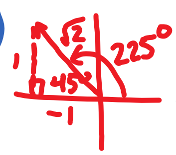
$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{(2)+(-\frac{1}{3})}{1-(2)(-\frac{1}{3})} = \dfrac{5/3}{5/3}$ $\boxed{=1}$

2. Find *exact* value.

a) $\dfrac{\tan 100^o + \tan 50^o}{1-\tan 100^o \tan 50^o} = \tan(\alpha+\beta)$
$= \tan(100^o + 50^o)$
$= \tan(150^o)$
$= -\dfrac{1}{\sqrt{3}}$
$\boxed{= -\dfrac{\sqrt{3}}{3}}$



b) $\dfrac{\tan\frac{4\pi}{3} - \tan\frac{\pi}{12}}{1+\tan\frac{4\pi}{3}\tan\frac{\pi}{12}} = \tan(\alpha-\beta)$
$= \tan\left(\frac{4\pi}{3} - \frac{\pi}{12}\right)$
$= \tan\left(\frac{16\pi}{12} - \frac{\pi}{12}\right)$
$= \tan\left(\frac{15\pi}{12}\right)$
$= \tan\left(\frac{5\pi}{4}\right)$
$\boxed{=1}$



c) $\tan 195^o = \tan(60^o + 135^o)$
$= \dfrac{\tan 60^o + \tan 135^o}{1-\tan 60^o \tan 135^o} = \dfrac{(\sqrt{3})+(-1)}{1-(\sqrt{3})(-1)}$
$= \dfrac{\sqrt{3}-1}{1+\sqrt{3}} \cdot \dfrac{(1-\sqrt{3})}{(1-\sqrt{3})} = \dfrac{\sqrt{3}-3-1+\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3}$
$= \dfrac{2\sqrt{3}-4}{-2} = \dfrac{2(\sqrt{3}-2)}{-2} = \boxed{-\sqrt{3}+2}$

d) $\tan 255^o = \tan(225^o + 30^o)$
$= \dfrac{(1) + \left(\frac{\sqrt{3}}{3}\right)}{1-(1)\left(\frac{\sqrt{3}}{3}\right)} = \dfrac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} \cdot \dfrac{\left(1+\frac{\sqrt{3}}{3}\right)}{\left(1+\frac{\sqrt{3}}{3}\right)}$
$= \dfrac{1+\frac{2\sqrt{3}}{3}+\frac{3}{9}}{1-\frac{3}{9}} = \dfrac{\frac{4}{3}+\frac{2\sqrt{3}}{3}}{\frac{2}{3}}$
$= \dfrac{4+2\sqrt{3}}{3} \cdot \dfrac{3}{2} = \dfrac{4+2\sqrt{3}}{2} = \boxed{2+\sqrt{3}}$

3. Evaluate $\tan\left(\frac{3\pi}{4} - \theta\right)$ when $\tan\theta = \frac{1}{3}$.

$\tan\left(\frac{3\pi}{4} - \theta\right) = \dfrac{\tan\frac{3\pi}{4} - \tan\theta}{1+\tan\frac{3\pi}{4}\tan\theta}$
$= \dfrac{(-1)-\left(\frac{1}{3}\right)}{1+(-1)\left(\frac{1}{3}\right)}$
$= \dfrac{-4/3}{2/3}$
$\boxed{=-2}$

$\tan\alpha = \frac{2}{3}$ $\tan\beta = \frac{2}{1}$

4. Suppose $\cot\alpha = \frac{3}{2}$ and $\cot\beta = \frac{1}{2}$. Find $\cot(\alpha - \beta)$.

$\cot(\alpha-\beta) = \dfrac{1}{\tan(\alpha-\beta)}$

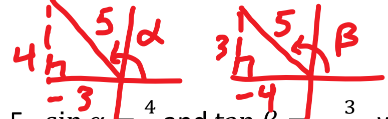
$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \dfrac{\left(\frac{2}{3}\right)-\left(\frac{2}{1}\right)}{1+\left(\frac{2}{3}\right)\left(\frac{2}{1}\right)}$

$\tan(\alpha-\beta) = \dfrac{-\frac{4}{3}}{\frac{7}{3}}$

$\tan(\alpha-\beta) = -\frac{4}{7}$

$\cot(\alpha-\beta) = \boxed{-\frac{7}{4}}$



5. $\sin\alpha = \frac{4}{5}$ and $\tan\beta = -\frac{3}{4}$, where $\frac{\pi}{2} < \alpha < \beta < \pi$. (Q2)

a) Find $\sin(\alpha + \beta)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(-\frac{4}{5}\right) + \left(-\frac{3}{5}\right)\left(\frac{3}{5}\right)$

$= -\frac{16}{25} - \frac{9}{25}$

$\boxed{= -1}$

b) $\cos(\alpha + \beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \frac{12}{25} - \frac{12}{25}$

$\boxed{= 0}$

c) $\tan(\alpha + \beta)$

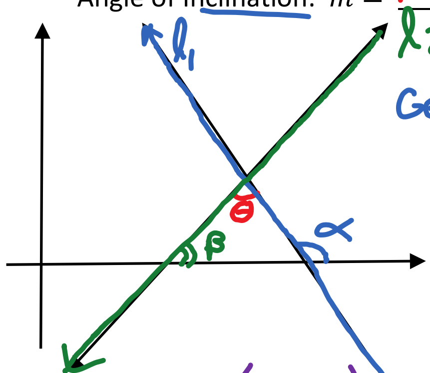
Fast way: $= \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{-1}{0}$ $\boxed{= \varnothing}$

Alternative: $= \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{\left(\frac{4}{-3}\right)+\left(\frac{-3}{4}\right)}{1-\left(-\frac{4}{3}\right)\left(-\frac{3}{4}\right)}$ $\boxed{= \varnothing}$

Angle Between two lines

Angle of Inclination: $m = \tan\alpha$, where $0 \le \alpha < 180^o$

inclination – the angle formed between the positive x-axis and a line ($0^\circ \le \alpha < 180^\circ$)



ext. ∠ = sum of 2 opp. int. ∠s

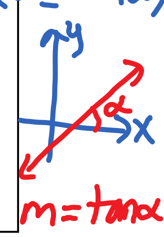
Geometry: $\alpha = \theta + \beta$

$\theta = \alpha - \beta$

Angle between two intersecting Lines

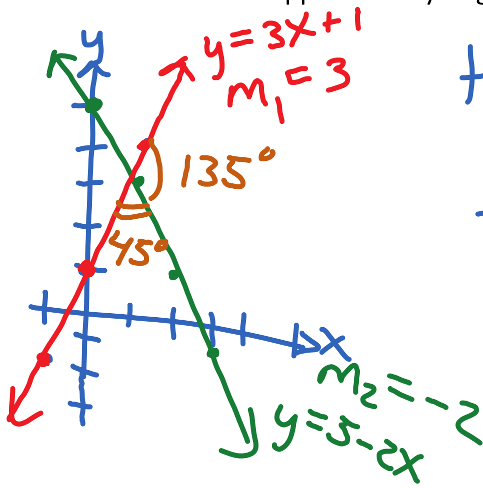
$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

↑ angle between 2 intersect lines



$\tan\theta = \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \dfrac{m_1 - m_2}{1+m_1 m_2}$

6. Find the 2 supplementary Angles formed between $y = 3x + 1$ and $y = 5 - 2x$.



$\tan\theta = \dfrac{m_1 - m_2}{1+m_1 m_2}$

$\tan\theta = \dfrac{3-(-2)}{1+(3)(-2)}$

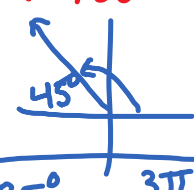
$\tan\theta = \dfrac{5}{-5}$

$\tan^{-1}(\tan\theta) = \tan^{-1}(-1)$

ref ∠ = 45°

* $0^\circ \le \alpha < 180^\circ$

Q2



Q2: $\boxed{135^\circ \text{ or } \frac{3\pi}{4}}$

supplementary angle $= 180^\circ - 135^\circ = \boxed{45^\circ \text{ or } \frac{\pi}{4}}$